Sorting

APS105: Computer Fundamentals

Jason Anderson

Dept. of Electrical and Computer Engineering
Faculty of Applied Science and Engineering
University of Toronto

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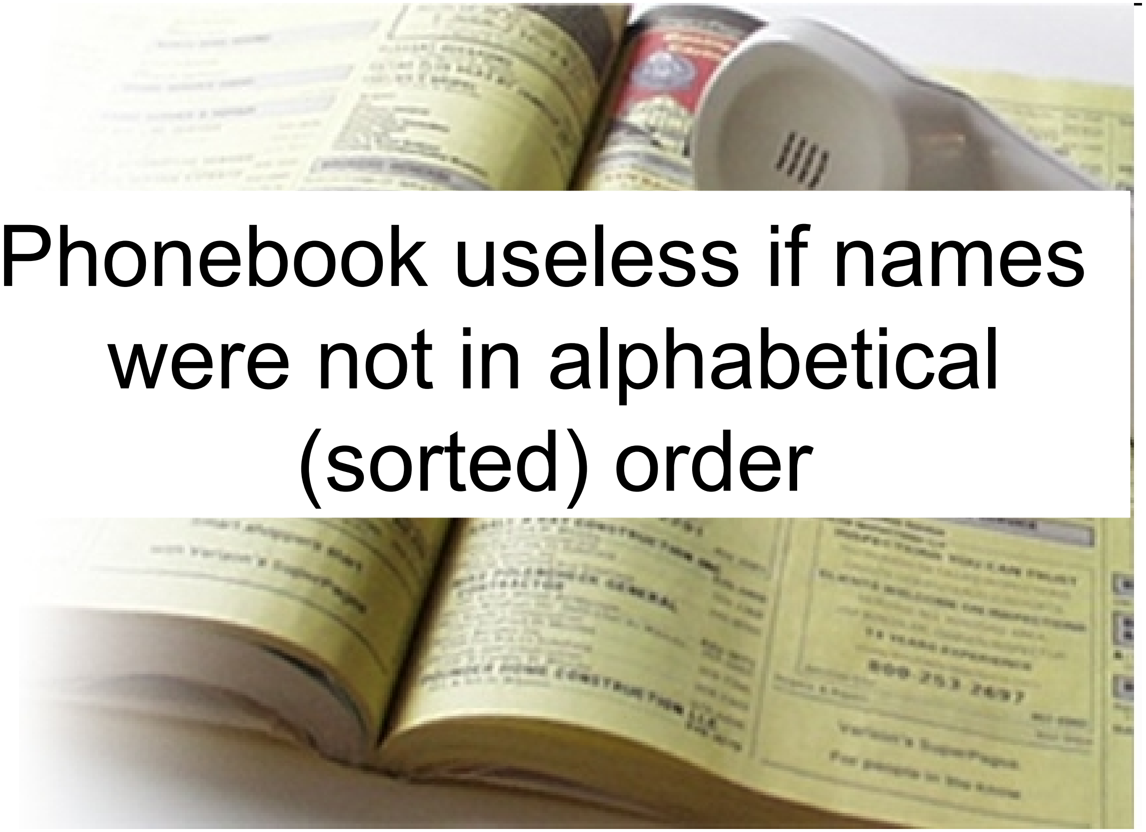
Sorting



Phonebook useless if names were not in alphabetical (sorted) order

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Why Sorting?

- Sorting used in **many** computer programs:
  - iPod sorts songs based on title or artist.
  - Facebook sorts friends in alphabetical order.
  - Facebook gives you the most-recent status.
  - Excel spreadsheet can sort column by values.
  - Online dating: sort dating matches based on how close by they live.
  - Youtube sorts based on popularity.
  - ….

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Sorting Objective

- Given: list of items (numbers, names, etc).
- Want to put the items sorted order.
    - Alpha order
    - Largest-to-smallest
    - Smallest-to-largest
    - Darkest colour to lightest colour
- Sometimes there are so many items, we need computer's help to do the sorting!

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What is an Algorithm?

- Set of steps for completing a task.
- You have **already** been designing algorithms in this class.
- You **already** use algorithms all the time in your daily life!

Algorithm for Baking a Cake

- Cream the butter.
- Mix the dry ingredients separately.
- Combine the dry and wet ingredients.
- Mix until smooth.
- Put into baking pan.
- Bake for 30 mins at 350º F.
- Is cake done?
  - If yes, remove from oven.
  - If no, bake for another 5 minutes.

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Algorithm Efficiency

- Washer, dryer take 30 mins each.
- Have one load of light clothes, one load dark clothes.

<u>Algorithm 1:</u>

2 pm : Light clothes into washer

2:30 pm: Light clothes into dryer

3 pm: Darks into washer

3:30 pm: Darks into dryer

All done at 4 pm!

<u>Algorithm 2:</u>

2 pm: Light clothes into washer

2:30 pm: Light clothes into dryer AND Darks into washer

3 pm: Darks into dryer

All done at 3:30 pm!

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Sorting Algorithms

- Sequence of steps computer takes for getting items into the right (sorted) order.
- Many different sorting algorithms invented:
  - But… they have different efficiencies!
    - Some take more “steps” to get things into the right order.
    - Some work better for different types of items.
- Want fast sorting algorithms:
  - Sort GTA phonebook with 6 million names.

Sorting Algorithms

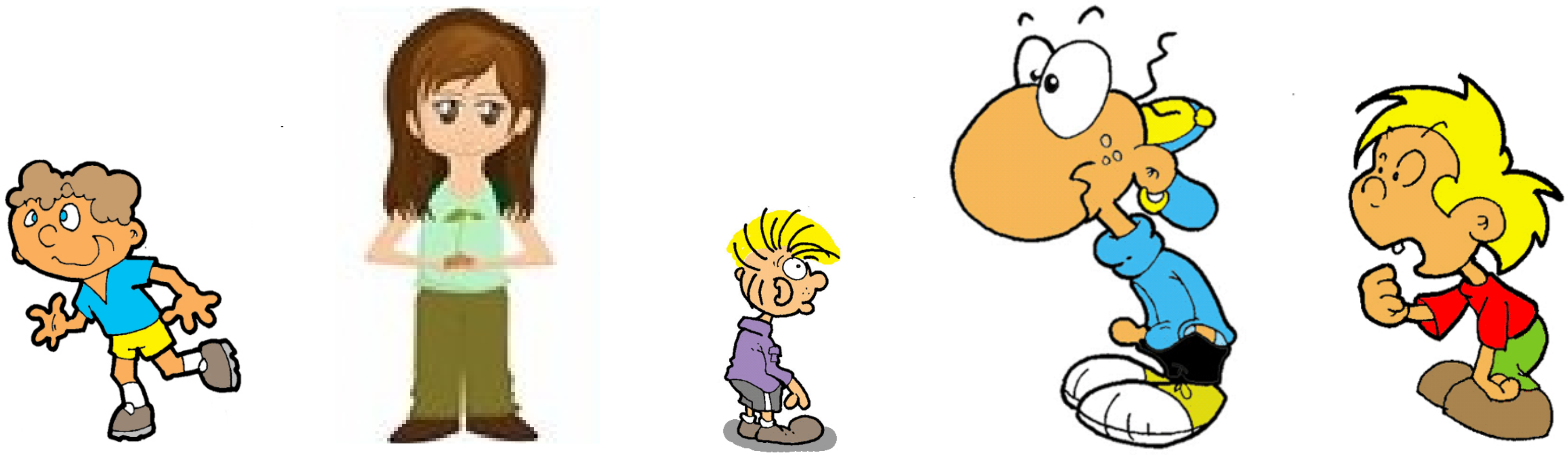
- Research on sorting algorithms:
  - Started in 1950s
  - Still active today
    (new algorithm invented in 2004)!
- We'll discuss three **classic** sorting approaches today:
  - Bubble sort
  - Insertion sort
  - Quicksort

Bubble Sort

- Walk through the list of numbers, comparing two items at a time.
  - Swap the two items if they're out of order.
- The biggest item "bubbles" up to the top.
- Walk the list of numbers several times until completely sorted!

Bubble Sort

Sort Children from Shortest to Tallest



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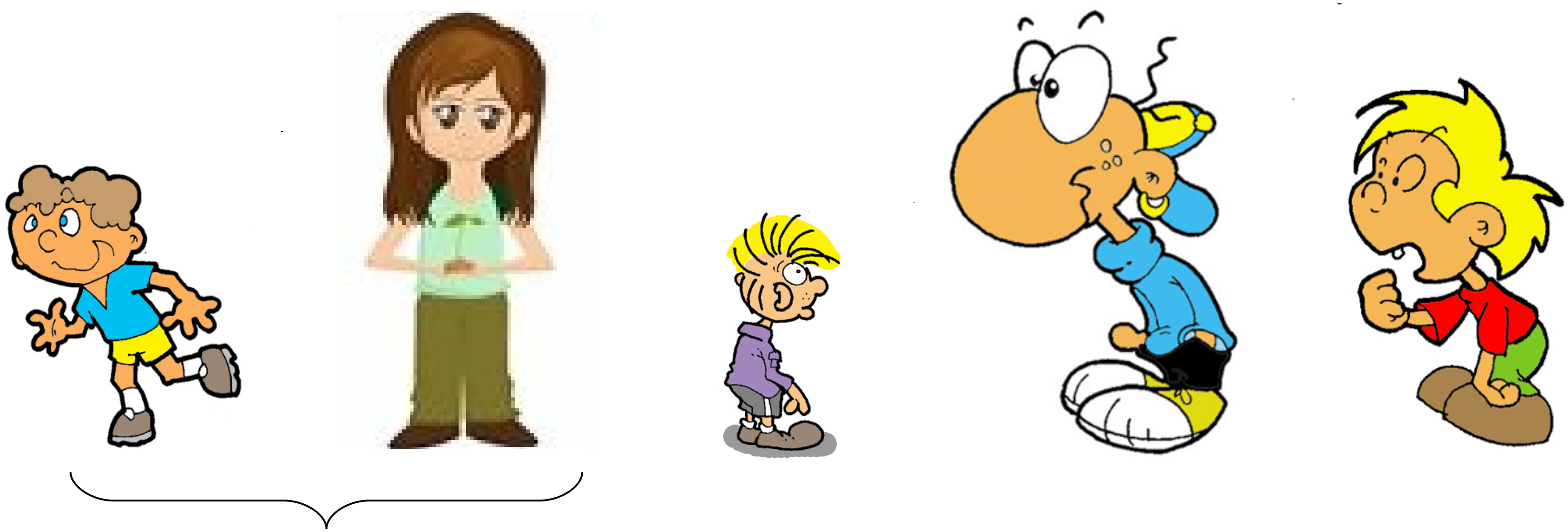

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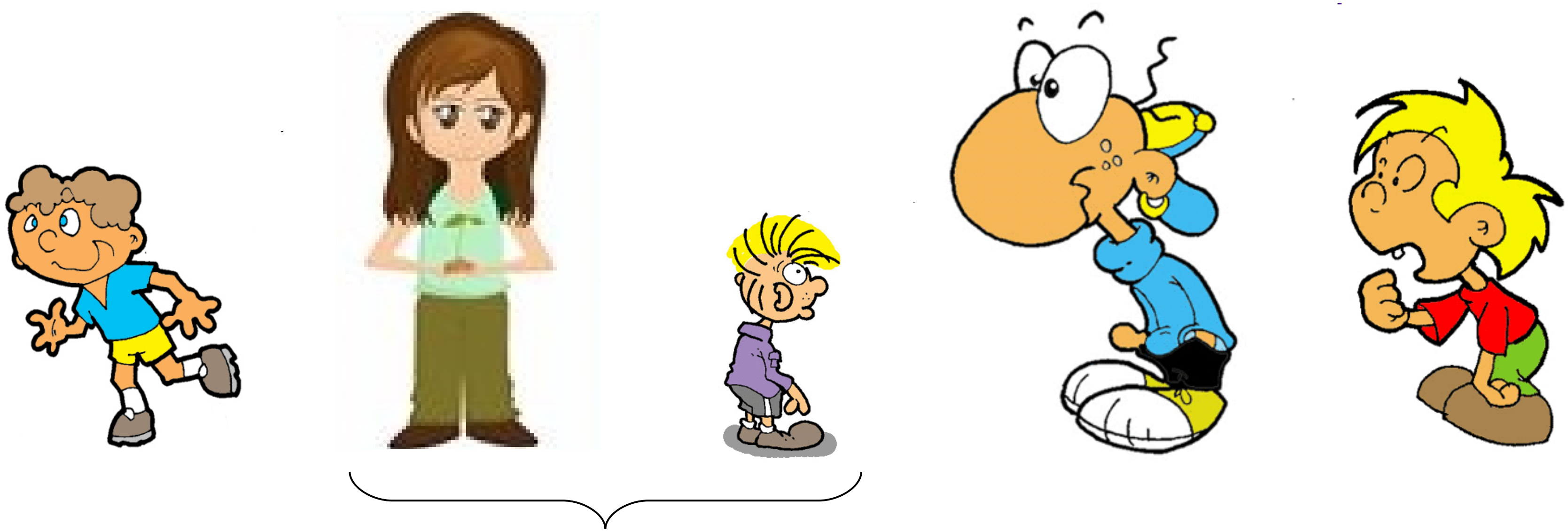

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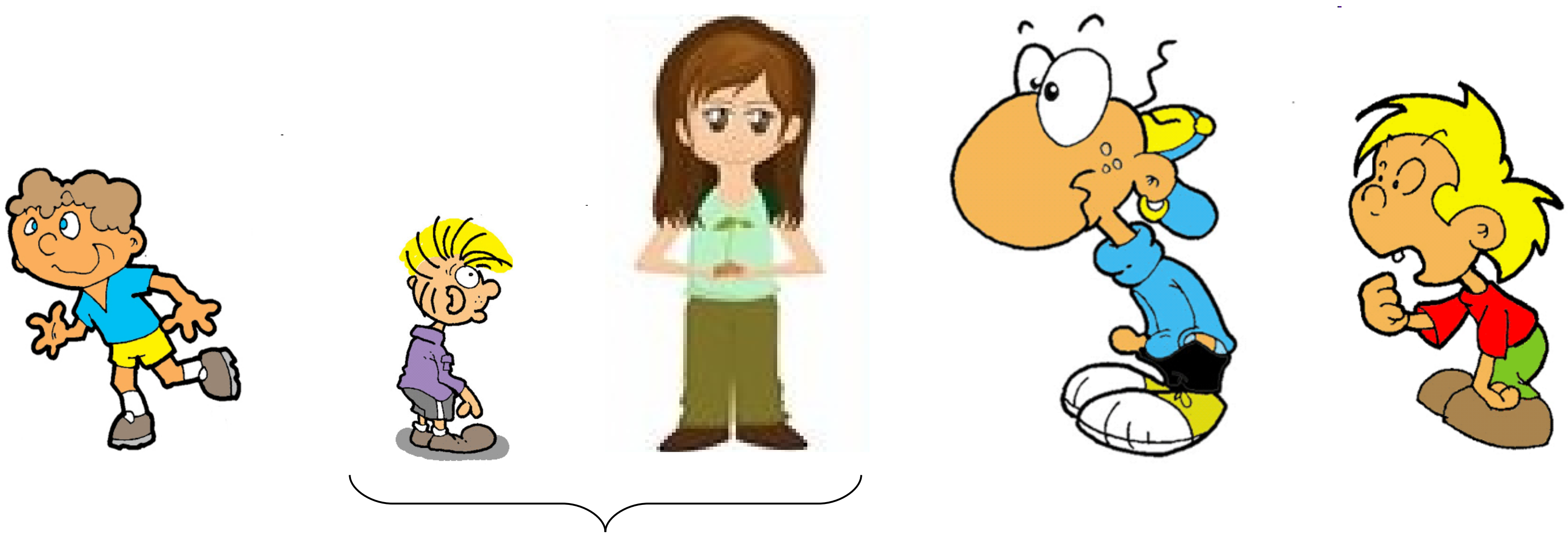

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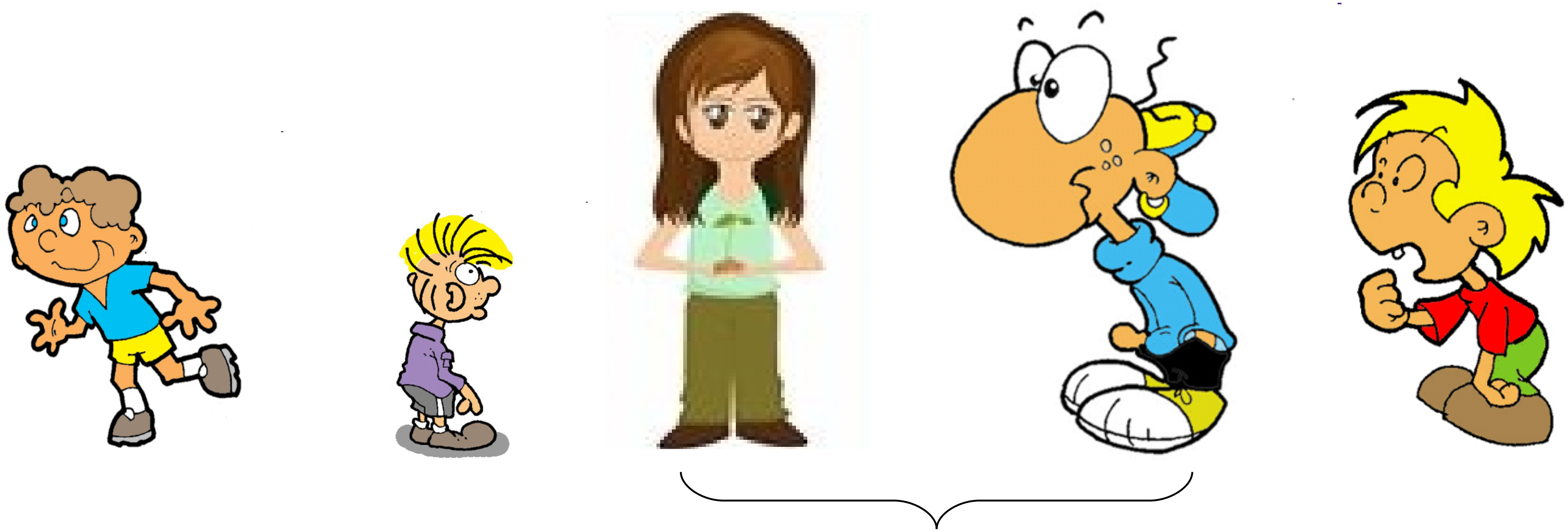

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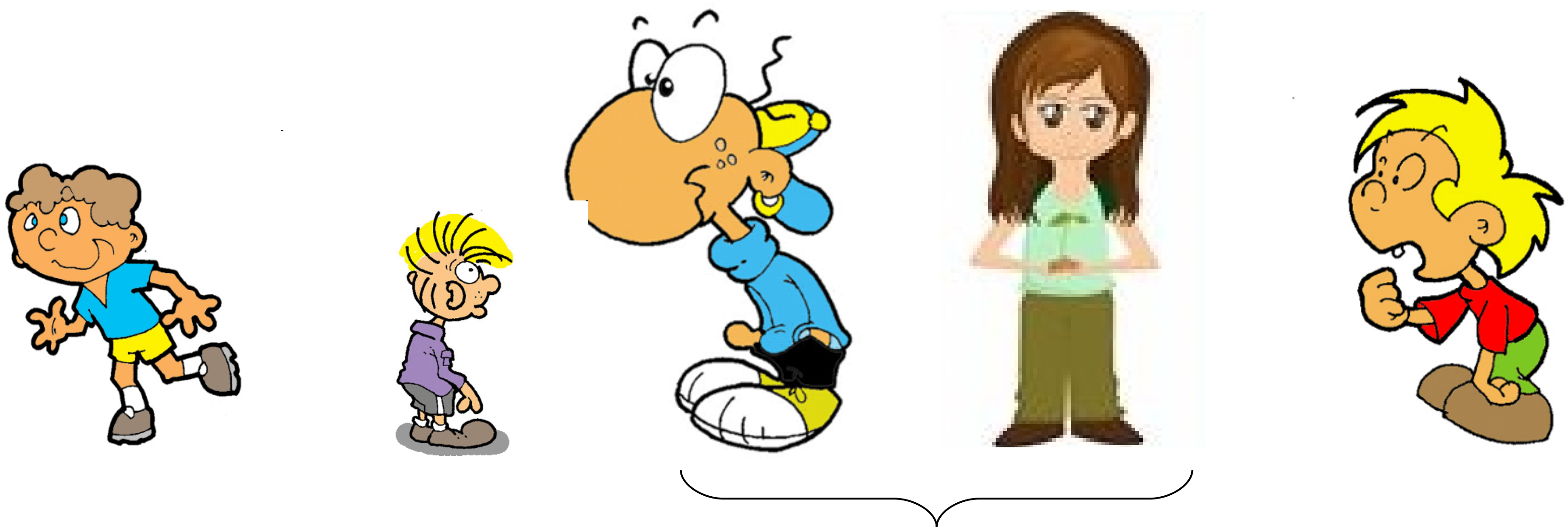

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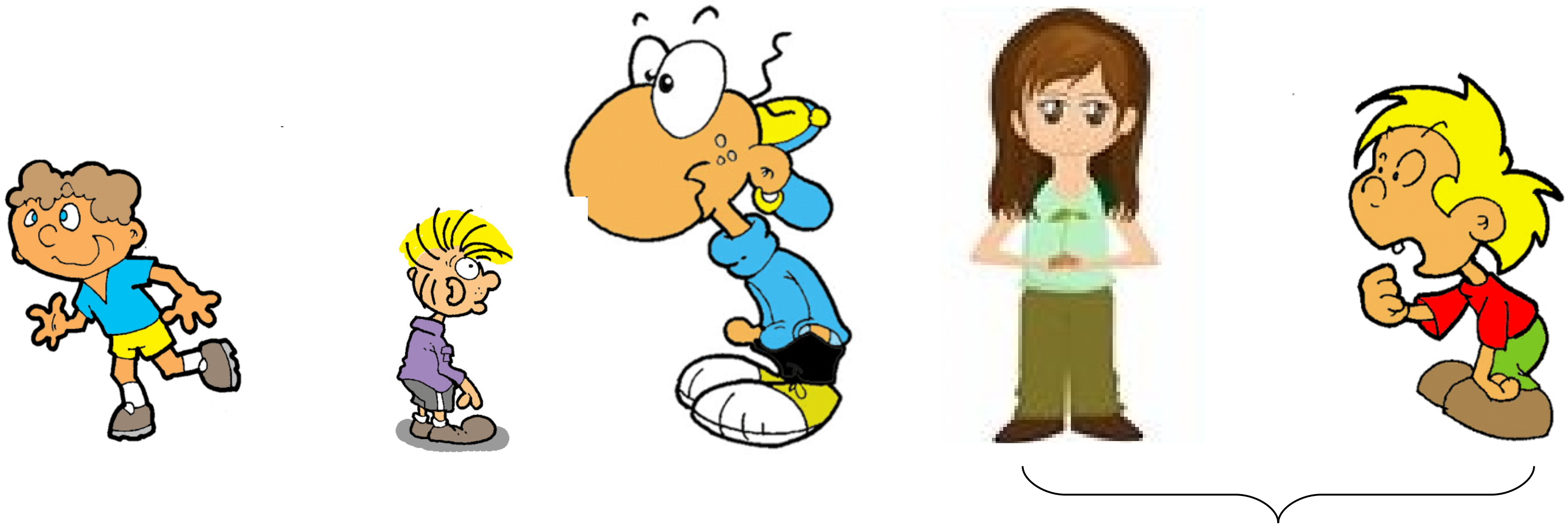

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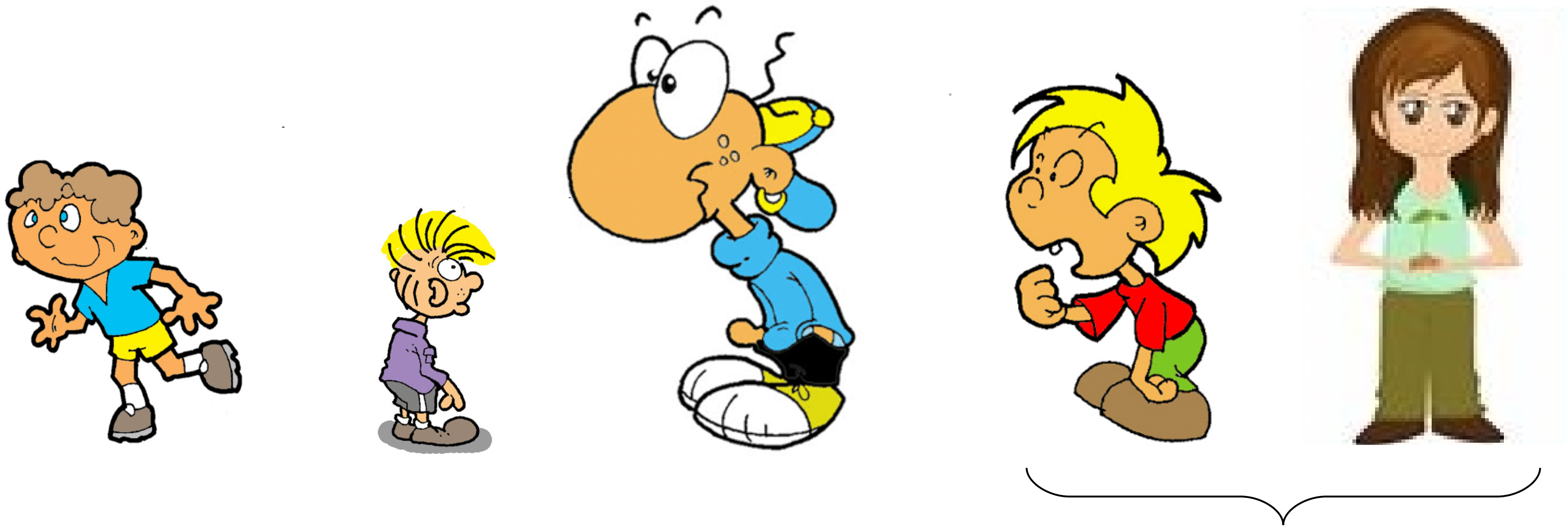

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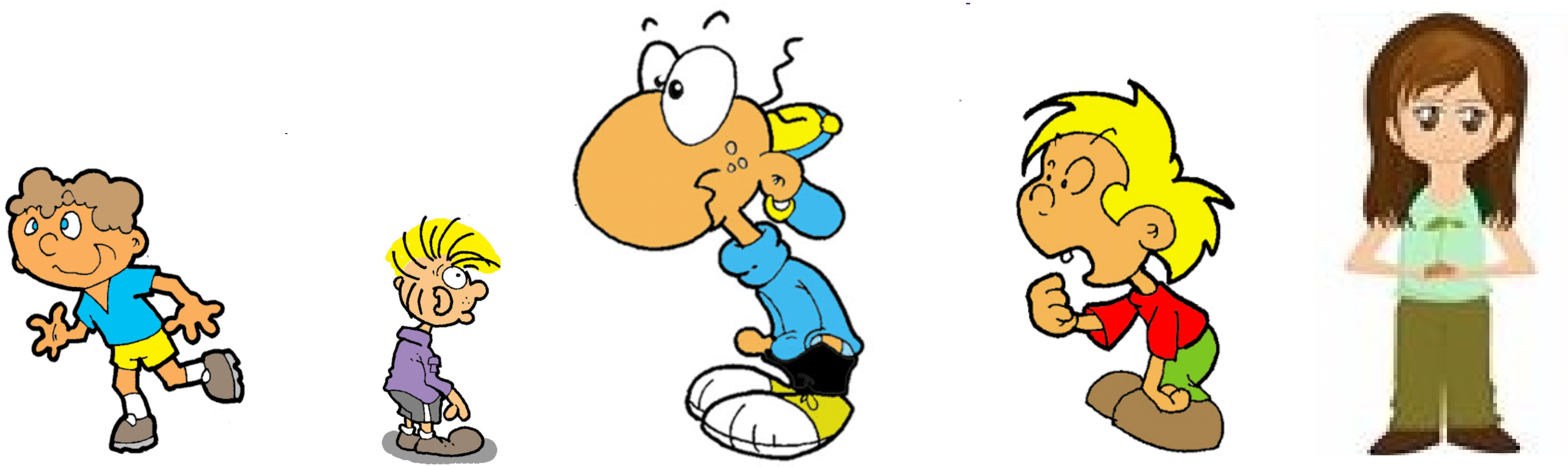

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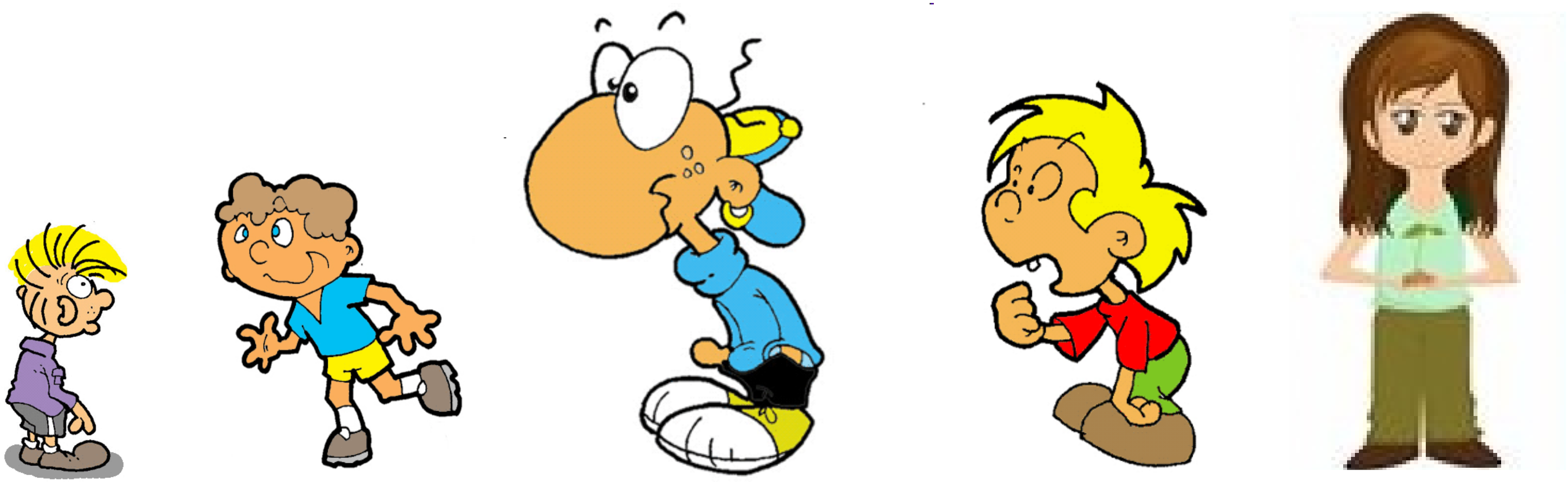

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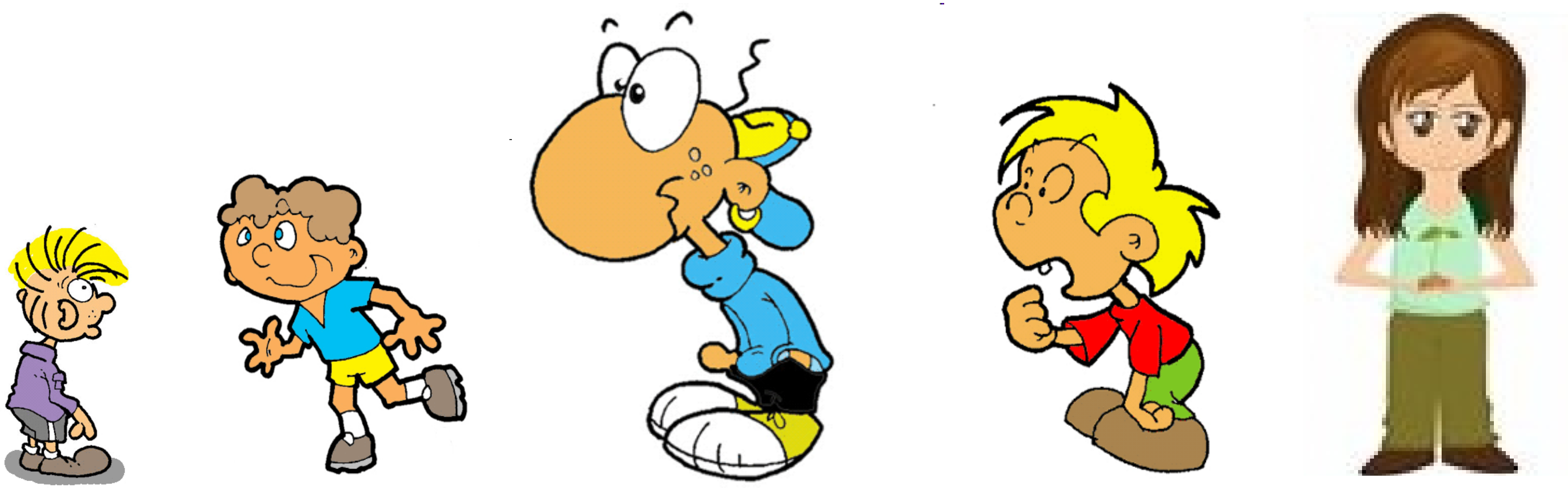

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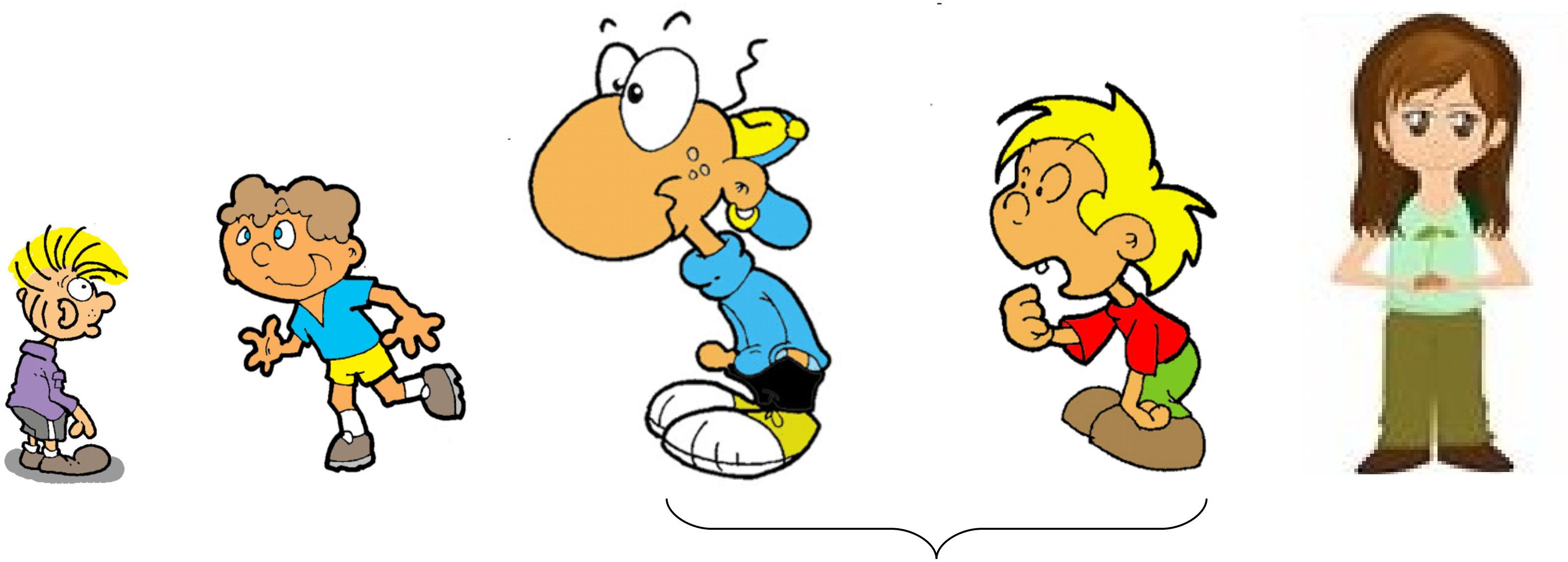

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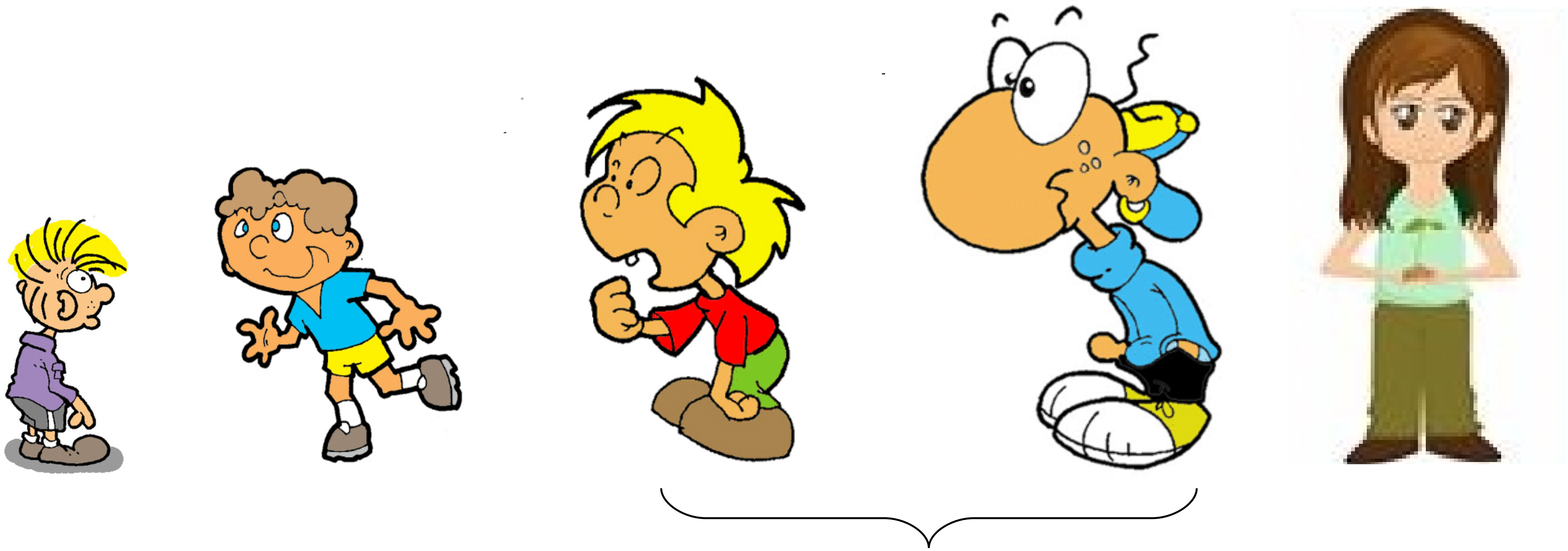

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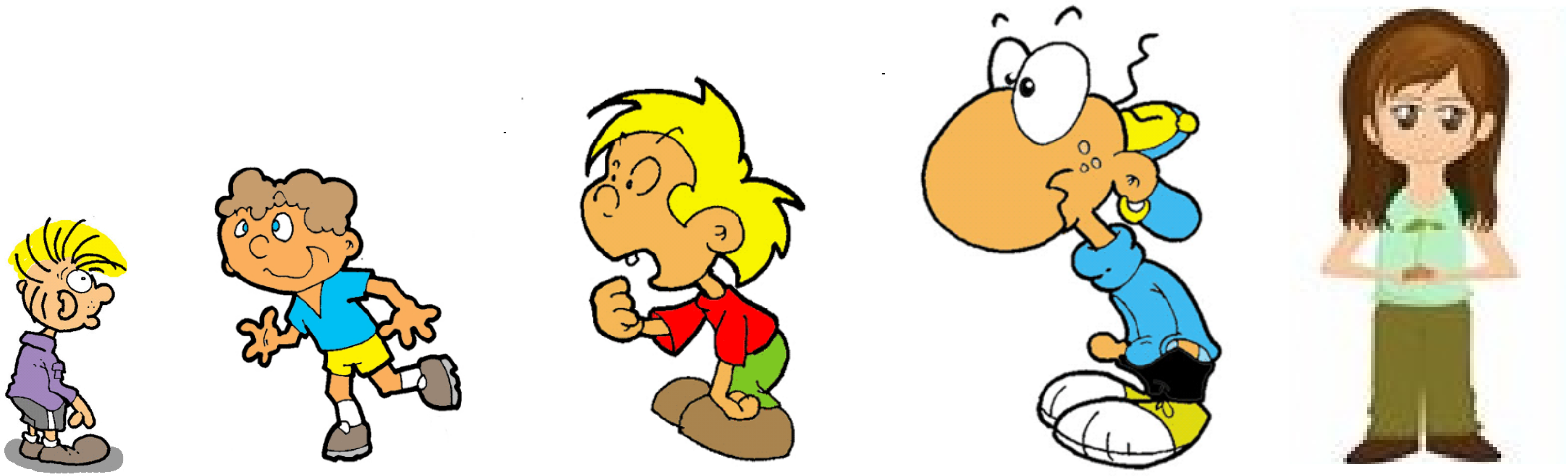

We don’t need to look at the last TWO, as in each pass the biggest “bubbles up” to the top spot.

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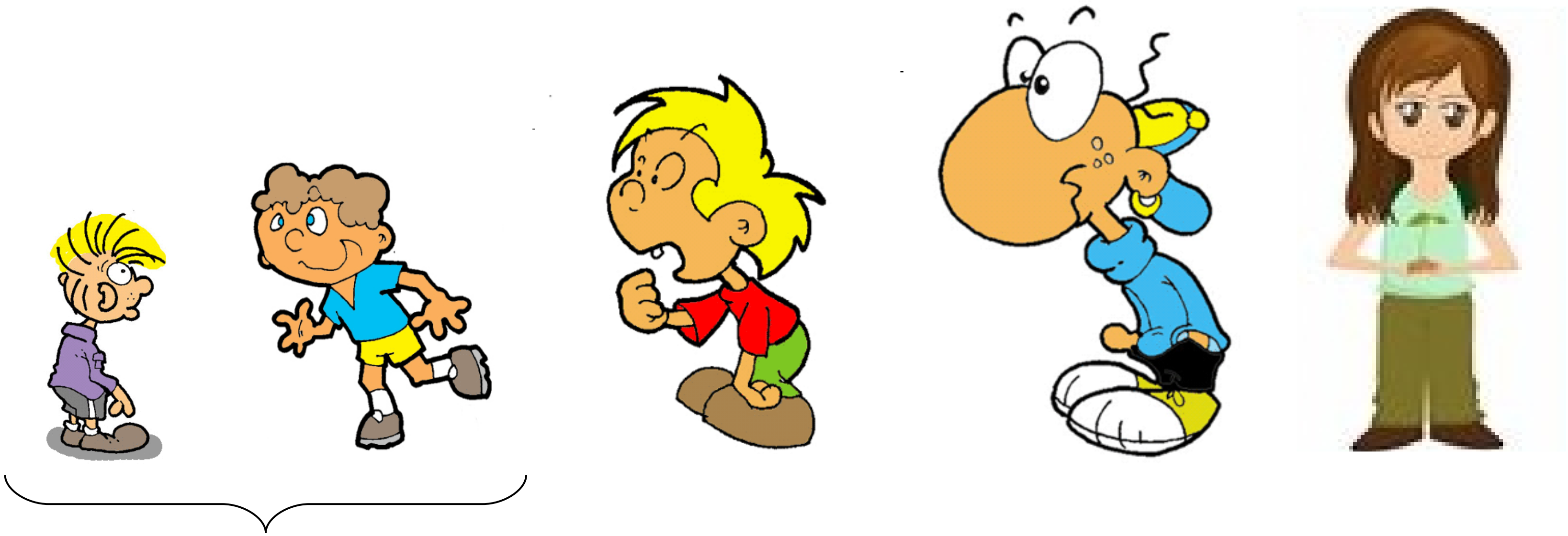

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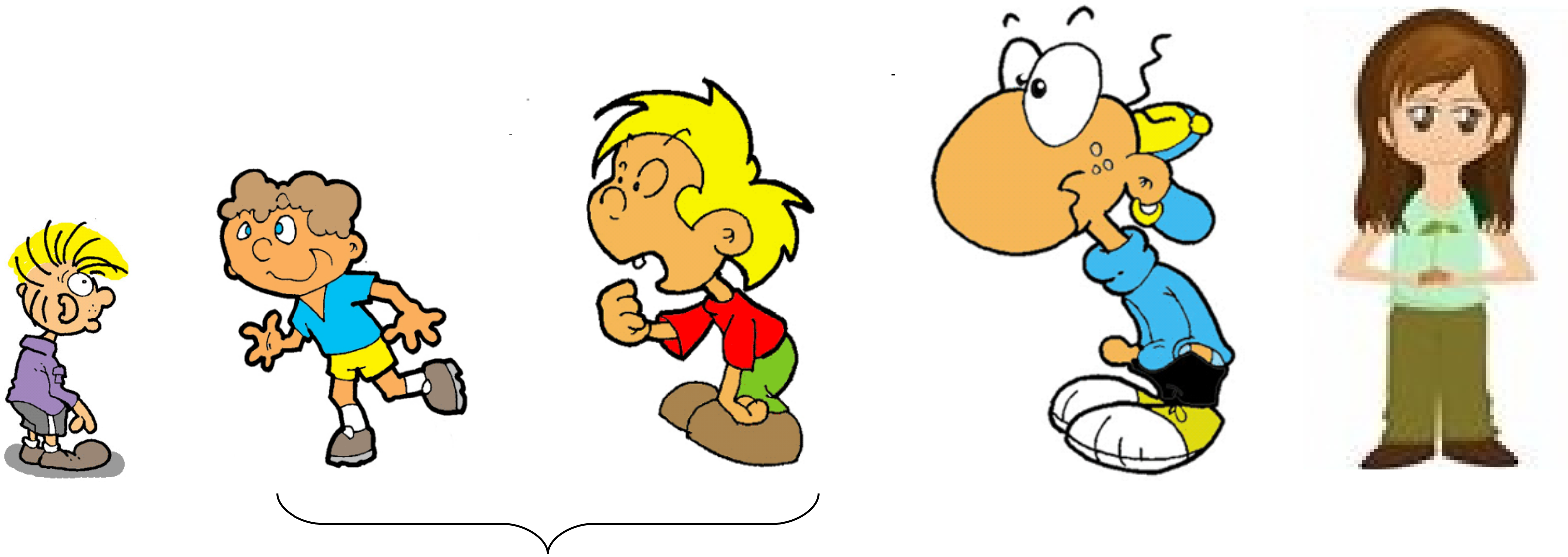

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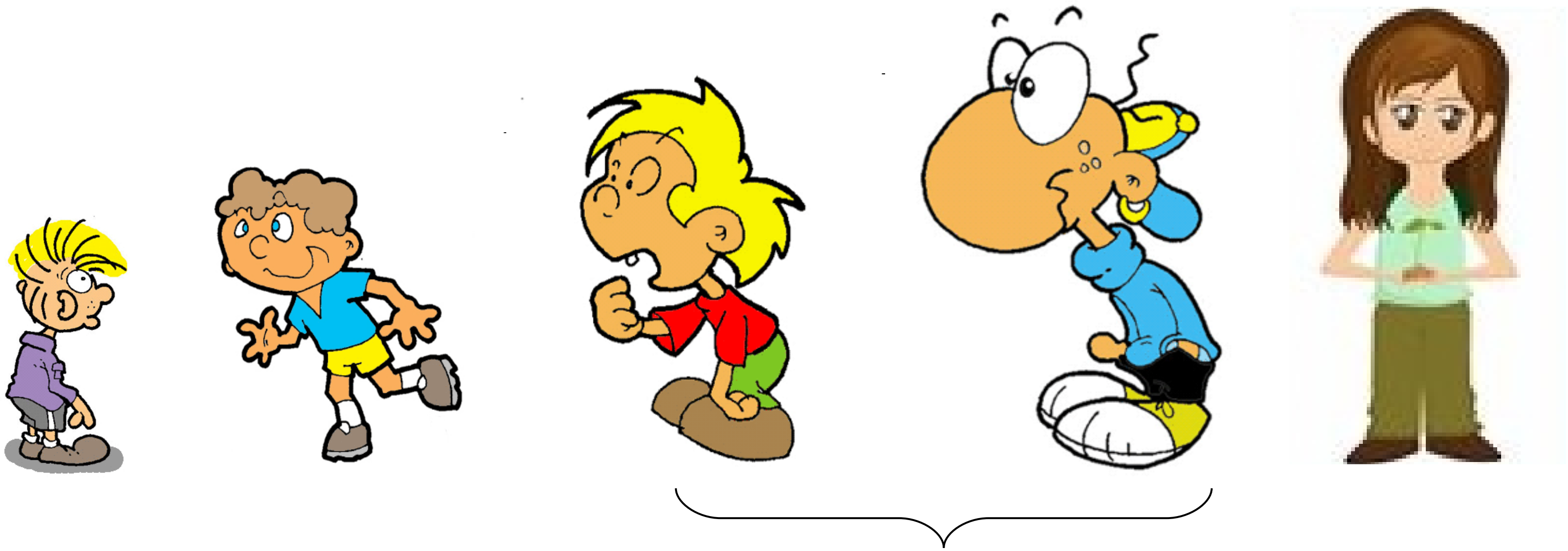

We don’t need to look at these TWO, as in each pass the biggest “bubbles up” to the top spot.

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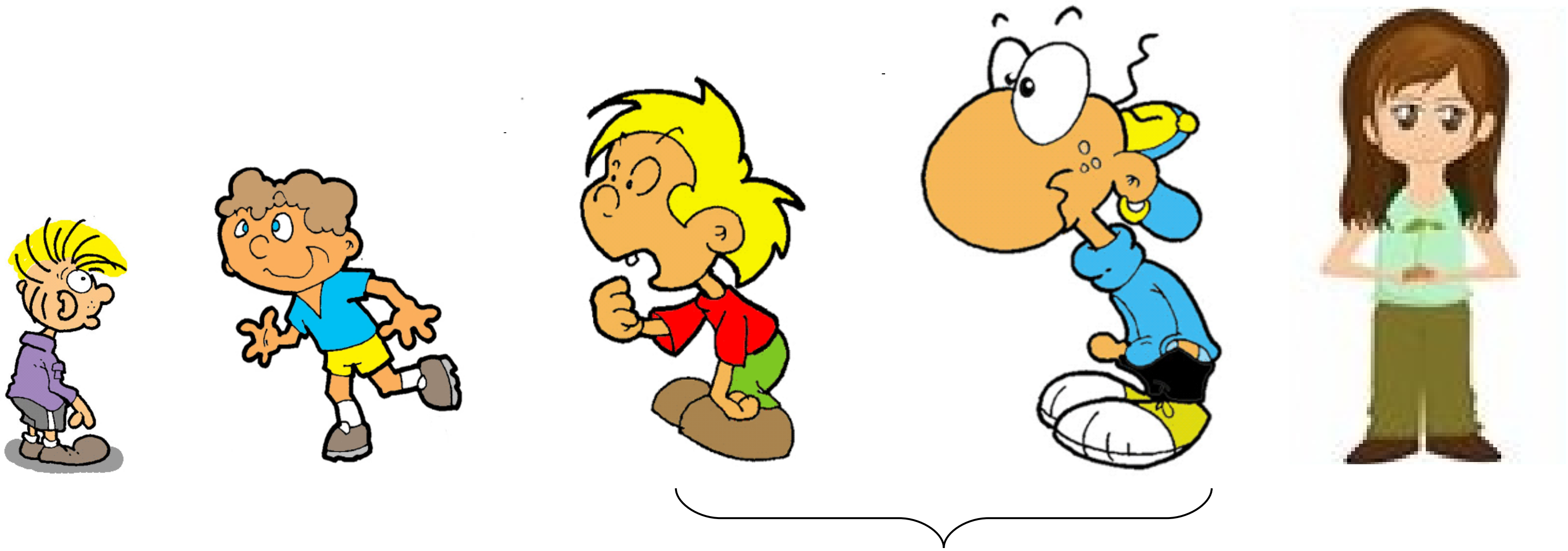

We don't need to look at these TWO, as in each pass the biggest "bubbles up" to the top spot.

Since we didn't make ANY swaps in this pass, we're DONE!

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Bubble Sort

- How many comparisons?
- Sort 1000 items:
  - 1st pass: 999 comparisons/swaps
  - 2nd pass: 998 comparisons/swaps
  - …
  - 999th pass: 1 comparison/swap
- Sort n items:
  - (n-1)*(n-2)/2 comparison/swaps

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Insertion Sort

- Like sorting playing cards in hand:
  - Draw a first card → card is sorted.
  - Draw second card → compare with first card.
  - Draw third card → compare with two cards.
  - …
  - …
  - Draw n^{th} card → compare with n-1 cards.
- Can **speed-up** using property that hand is kept sorted.

Quicksort

- Invented in 1962 by C. Hoare.
- **Much** more efficient than previous algorithms.
  - Fewer steps needed to get items into order.
- Still widely used today.
- Uses **recursion**.

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Recursion

Contains smaller version of same picture

Recursion in Computing

- Task is broken down into smaller/easier tasks solved in a similar way.
  - Ends when we hit a BASE/END case.

Recursion in Computing

- Task is broken down into smaller/easier tasks solved in a similar way.
  - Ends when we hit a BASE/END case.
  - Invest $1000 at 8% interest.
  - How much do I have in 3 years?
    - Cash(3 years) = Cash(2 years) + 8% x Cash(2 years)
    - Cash(2 years) = Cash(1 years) + 8% x Cash(1 years)
    - Cash(1 year) = Cash(0 years) + 8% x Cash(0 years)
    - Cash(0 years) = $1000.00

    - Cash(3 years) = $1259.71!

Quicksort Core Action

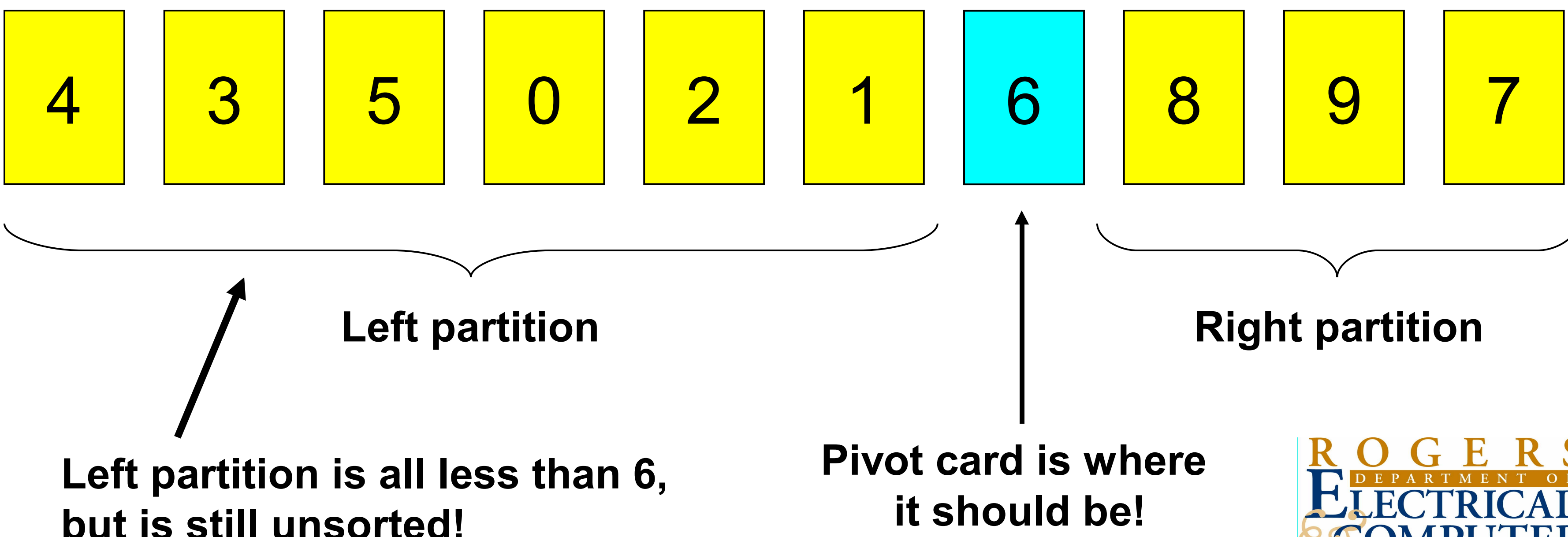
- Want to sort 10 cards:

6 3 9 0 2 1 7 4 5 8

- Choose a "pivot" card: 6

Quicksort Core Action

- Walk along the cards and "**partition**" the cards into two groups:
  - Cards smaller than the pivot.
  - Cards larger than the pivot.



Quicksort Core Action

- Now take the left partition & repeat action!

4 3 5 0 2 1

- Choose a "pivot" card: 4

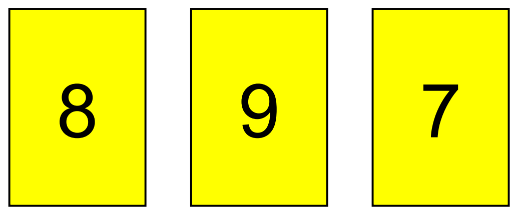
Pivot is in its correct place

- Partition the left partition:

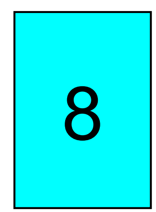
2 3 1 0 4 5

Quicksort Core Action

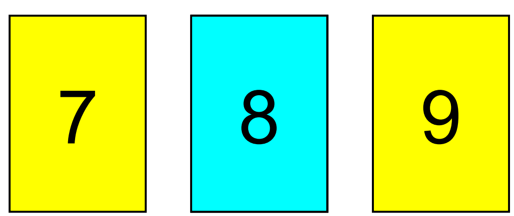
- Likewise, take the right partition and repeat the process.



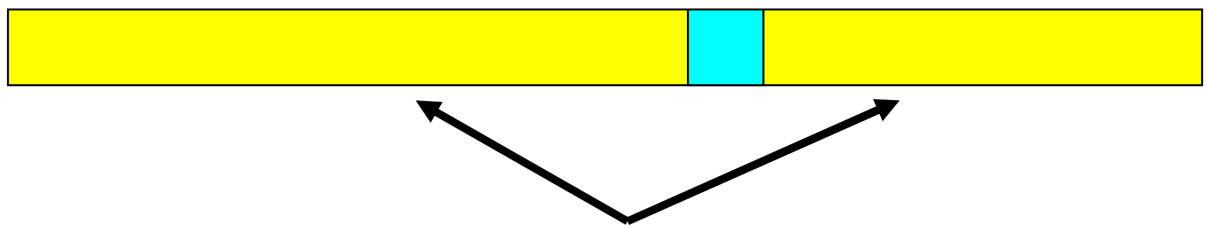
- Choose a “pivot” card:



- Partition the right partition:



Quicksort Overview



Pivot and partition the left and right half.

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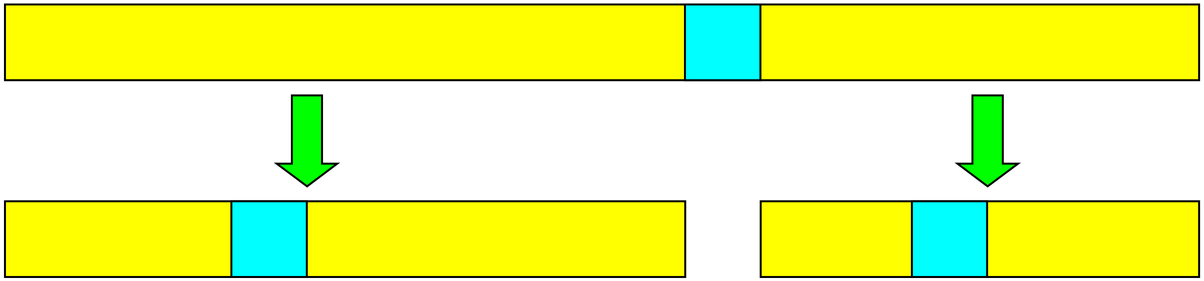

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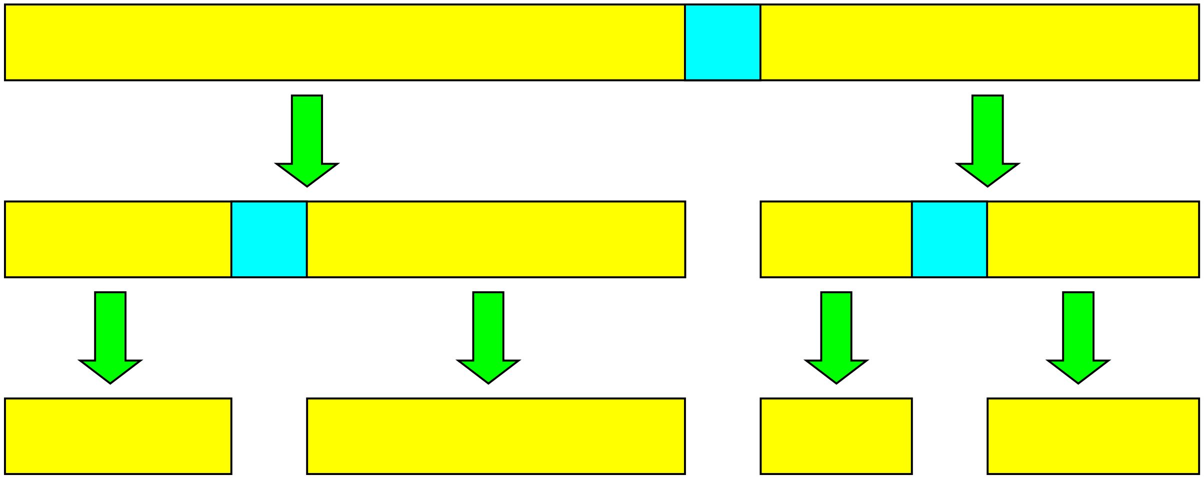

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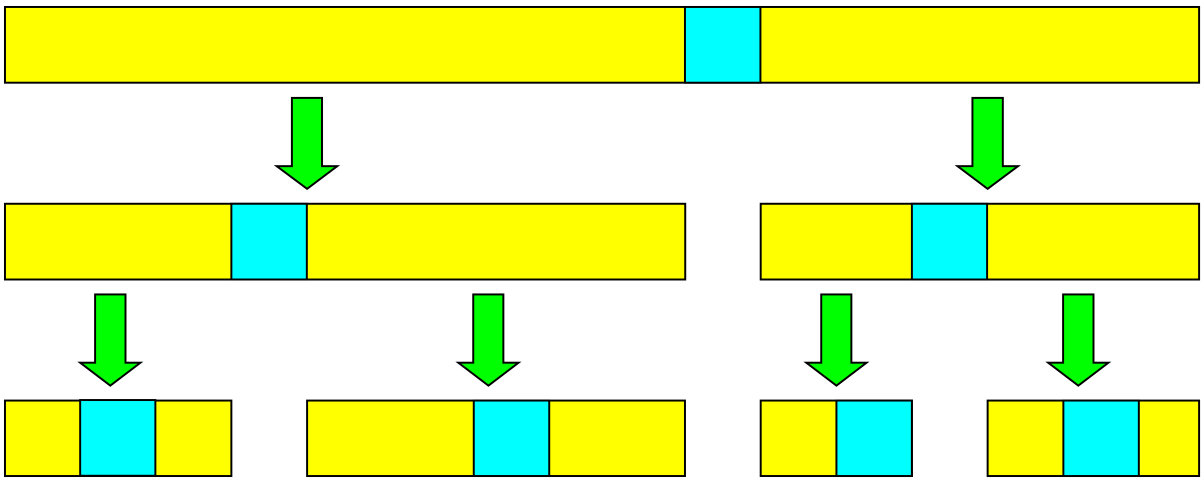

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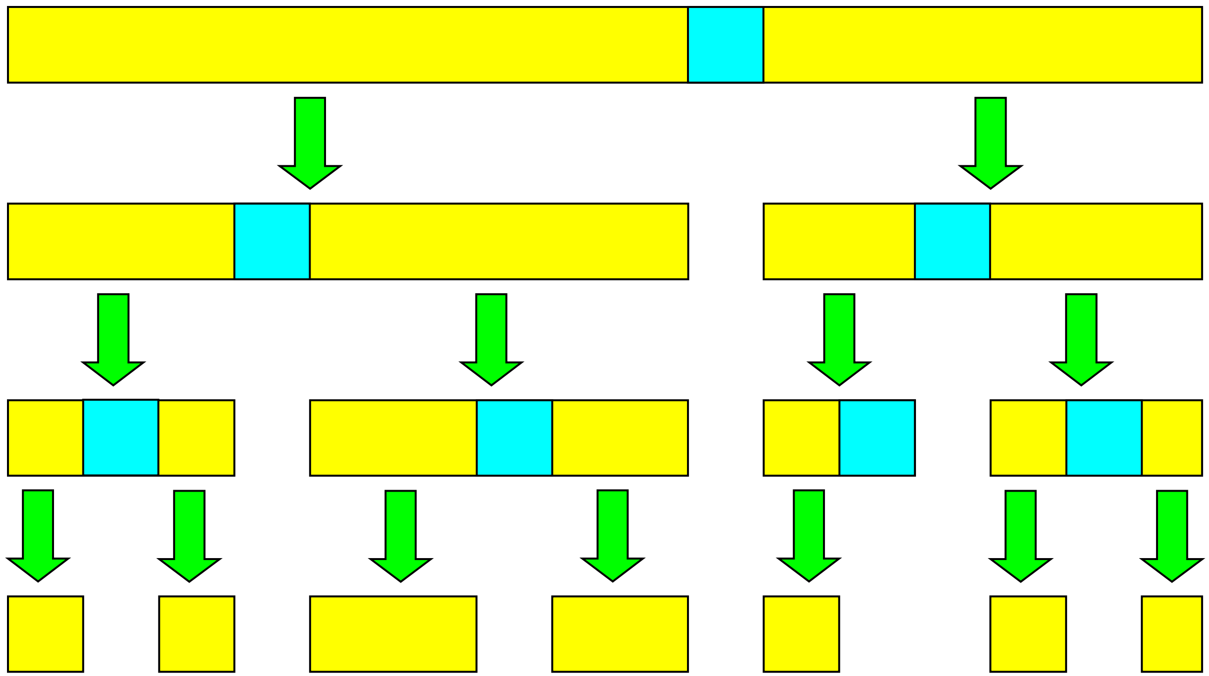

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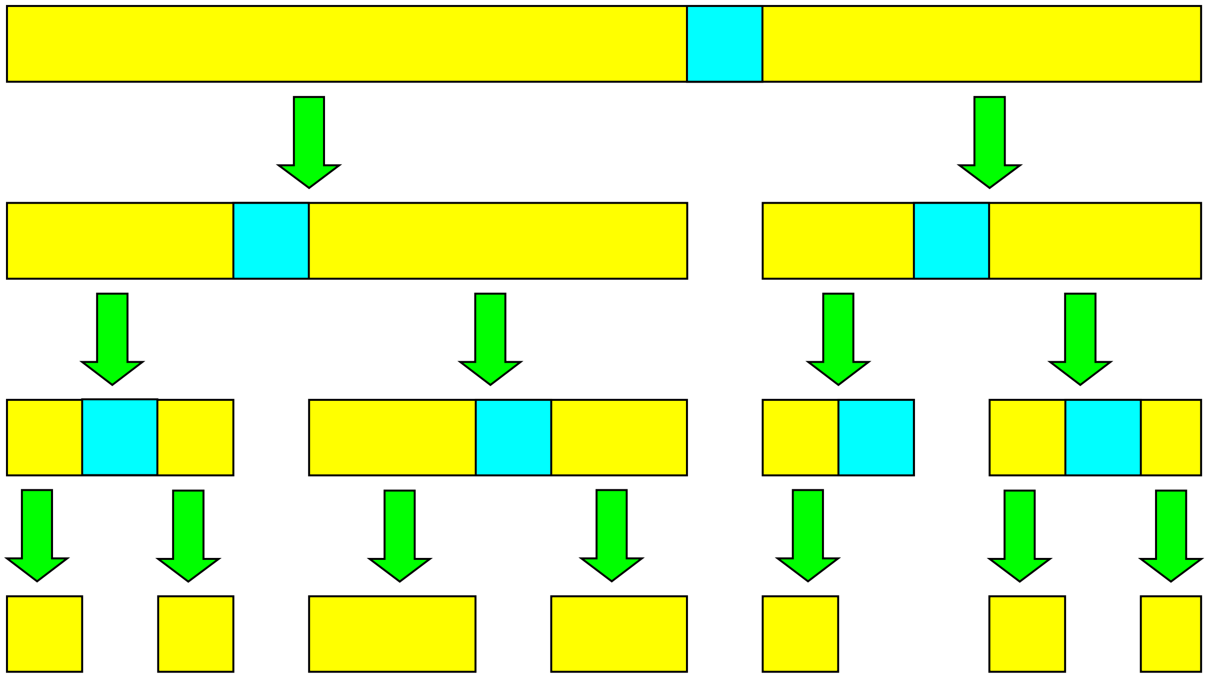
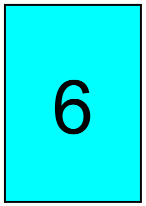
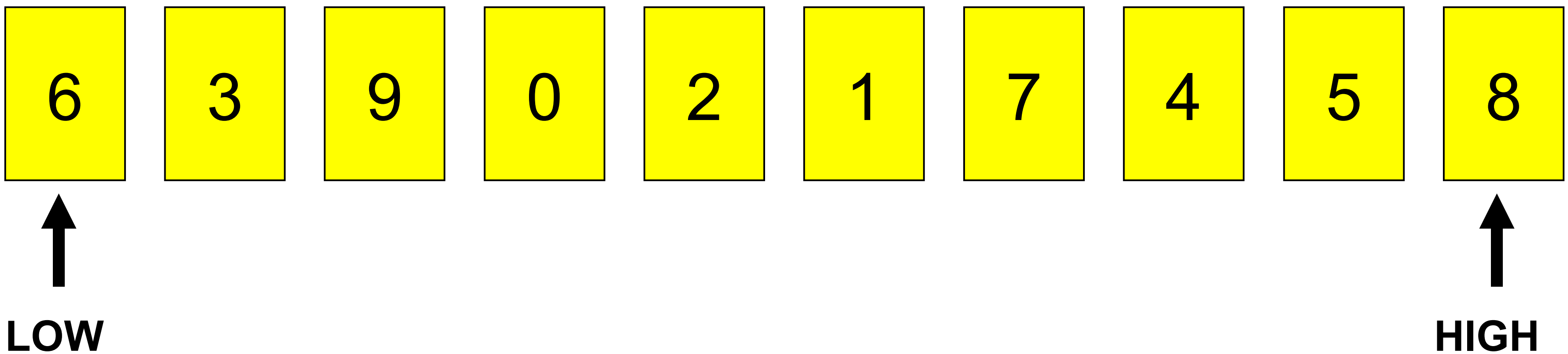


Continue until the pieces are so small there is nothing to do! $\rightarrow$ SORTED

Nitty Gritty Details

- How do we do the partitioning?





- Consider two “arrows” LOW and HIGH.

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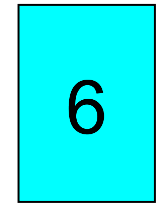

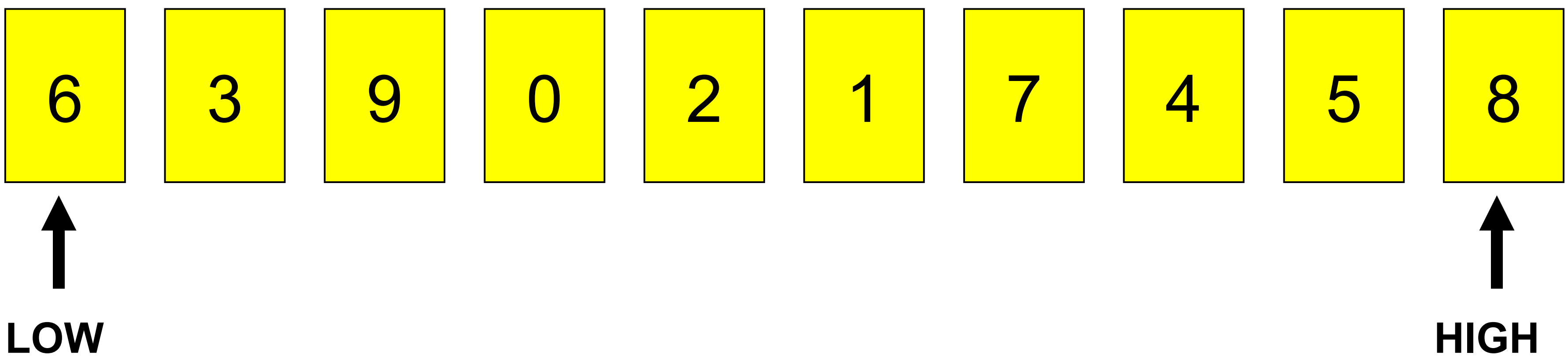

- Consider two “arrows” LOW and HIGH.
- HIGH will march LEFT until it finds a number less than PIVOT.

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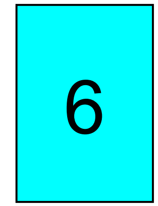

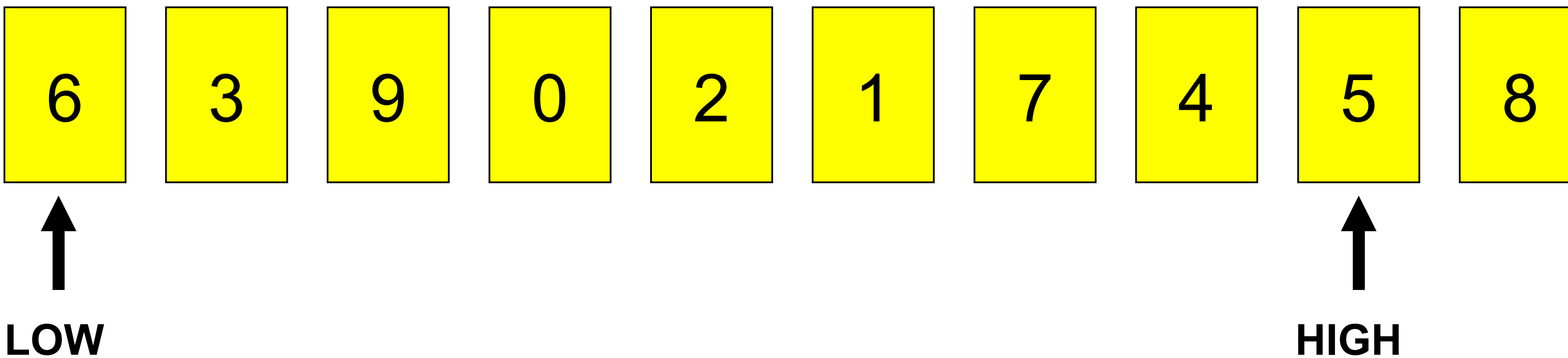

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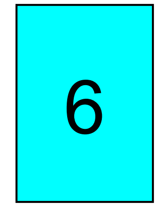

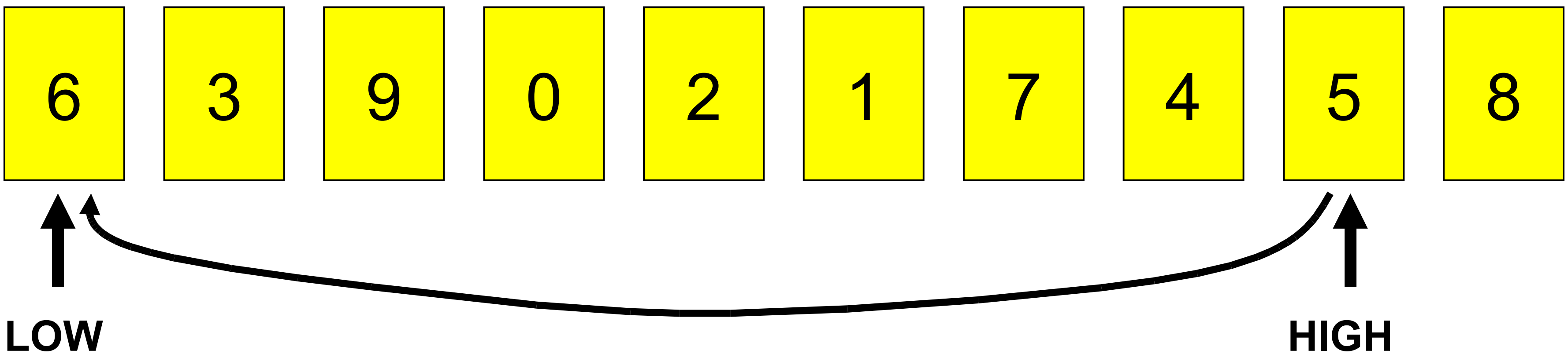

- The number at HIGH is moved to the position LOW.

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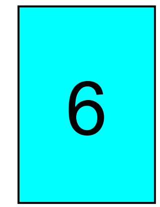

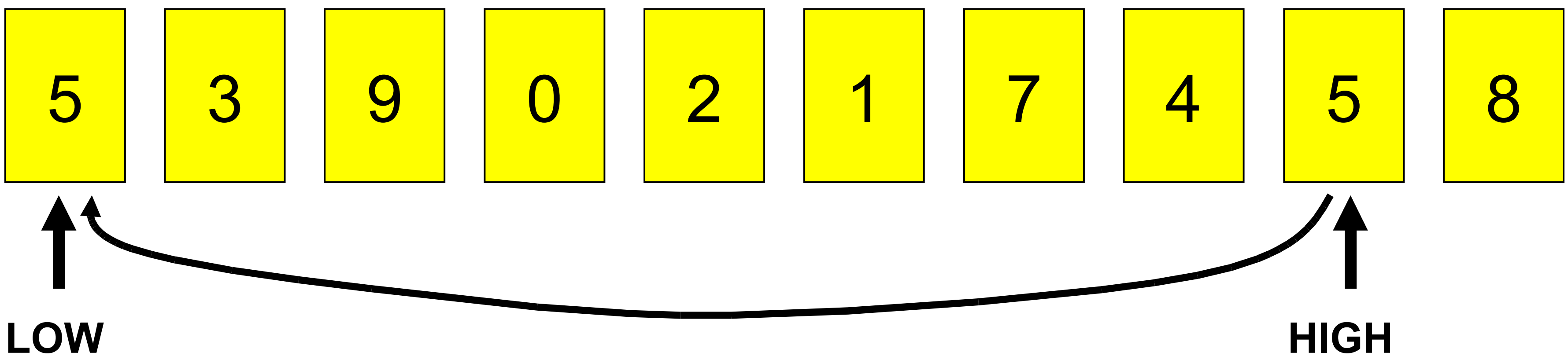

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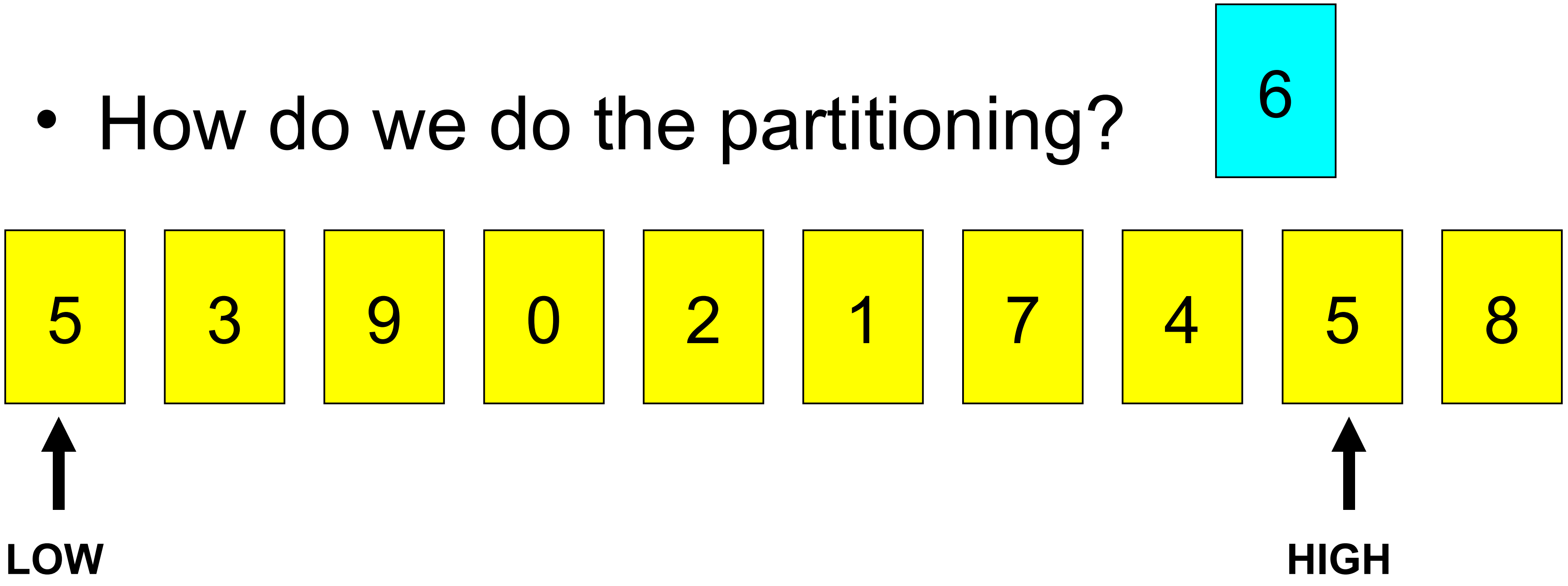

- LOW marches RIGHT until it finds a number greater than PIVOT

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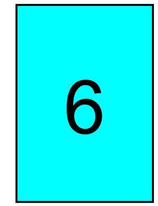

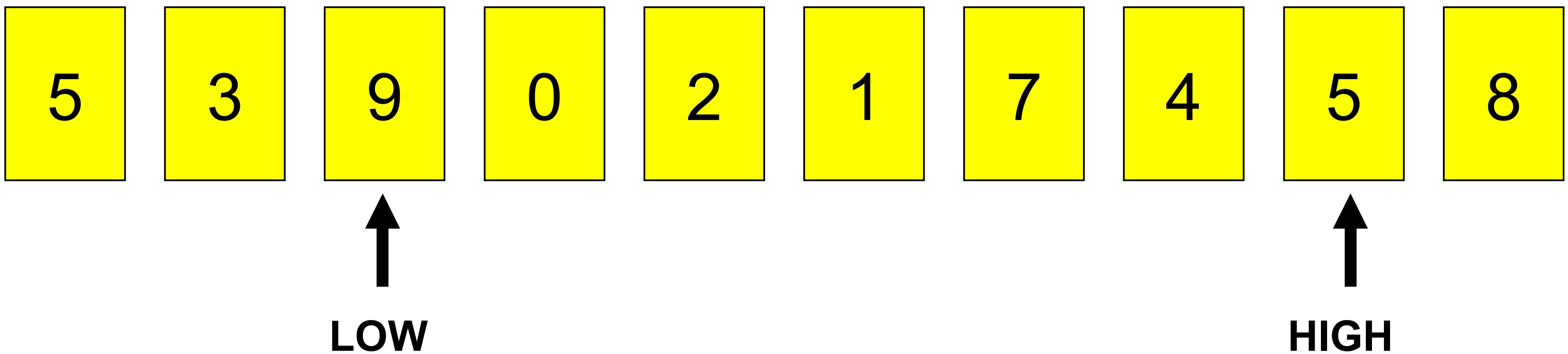

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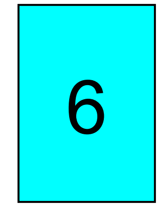

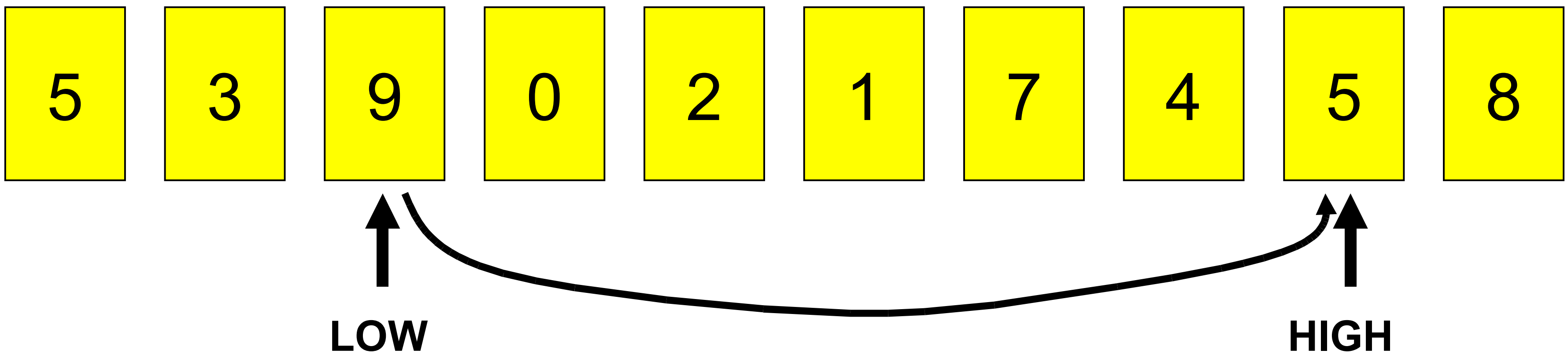

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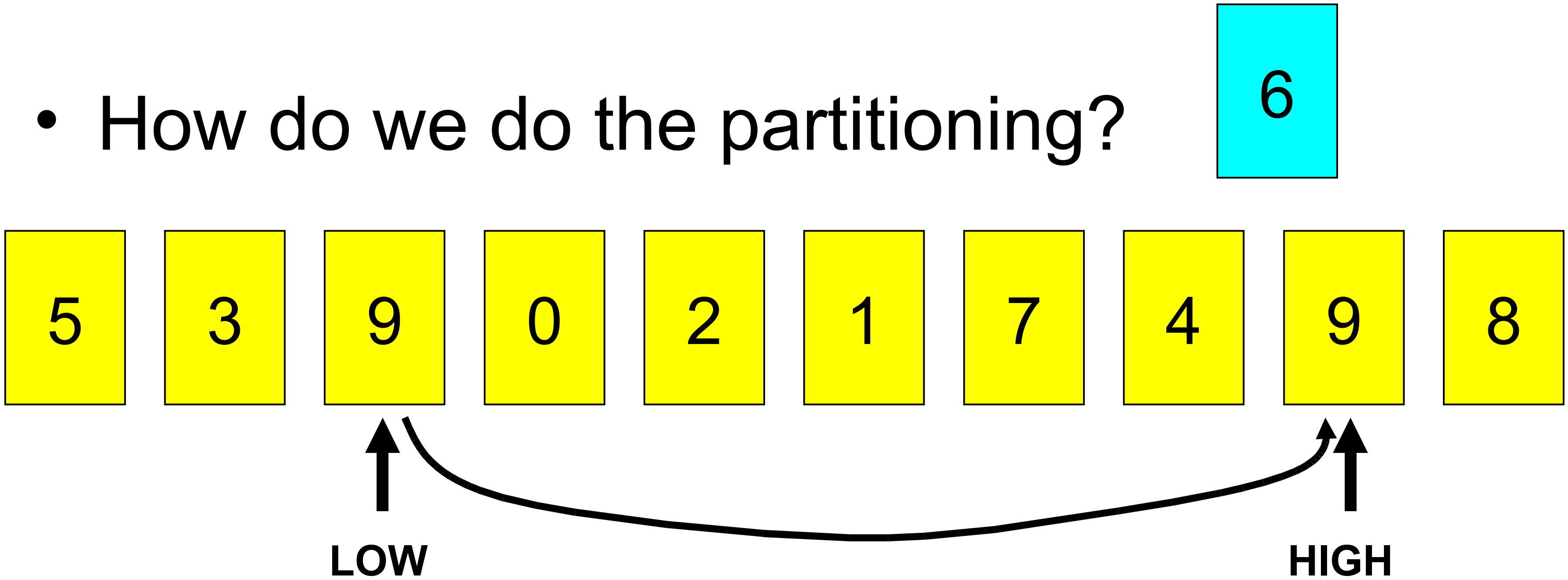

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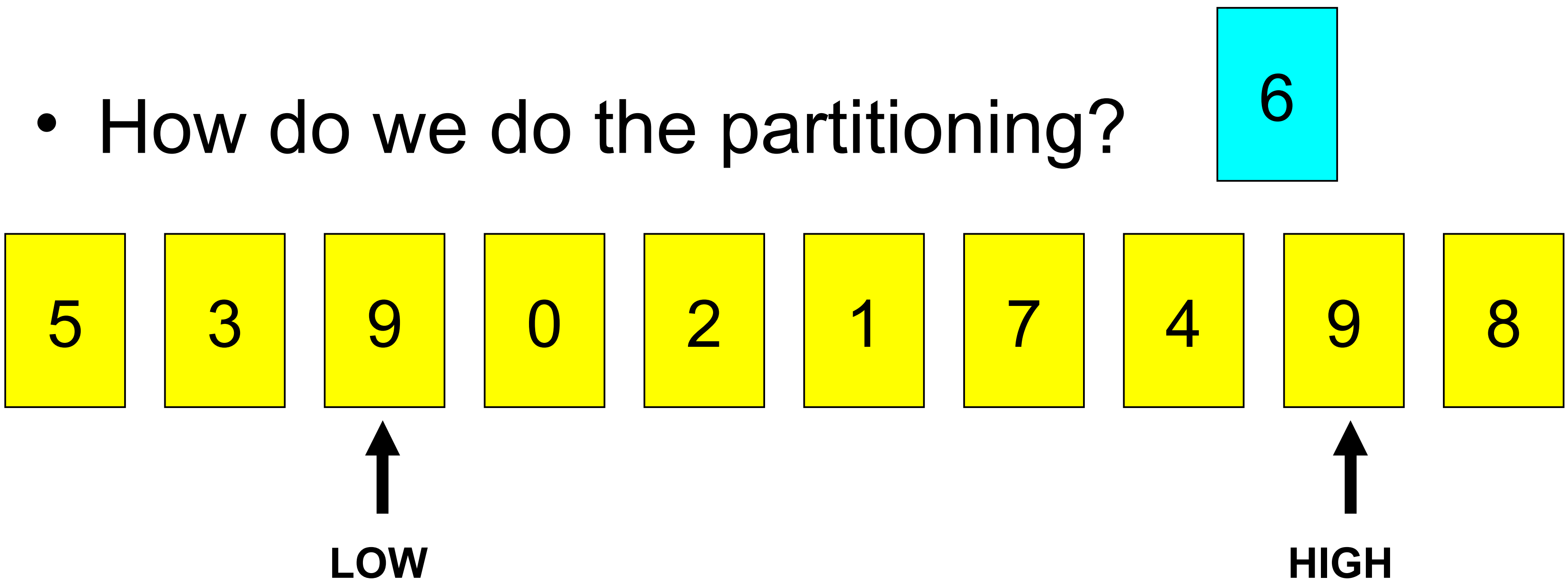

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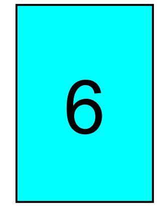

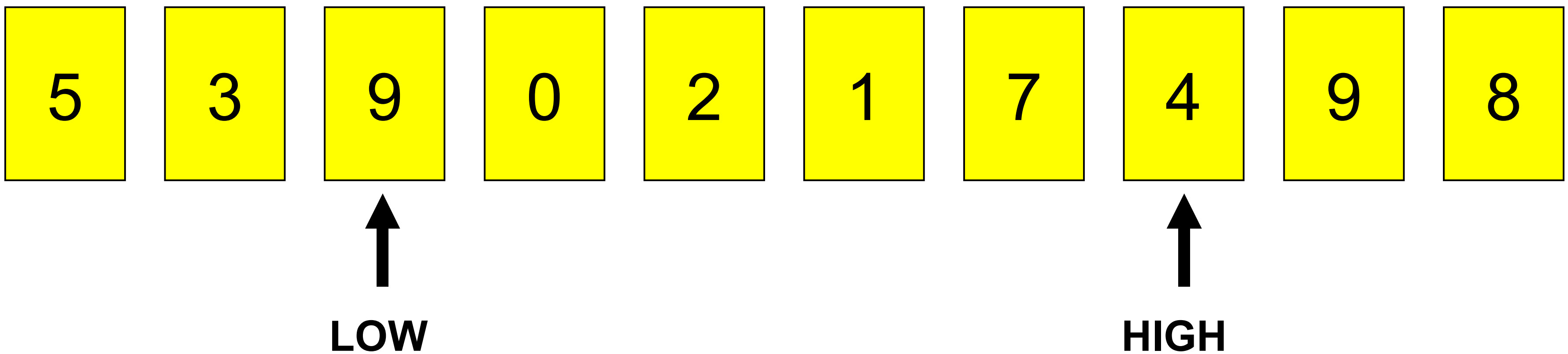

- HIGH marches left until it finds a number less than PIVOT.

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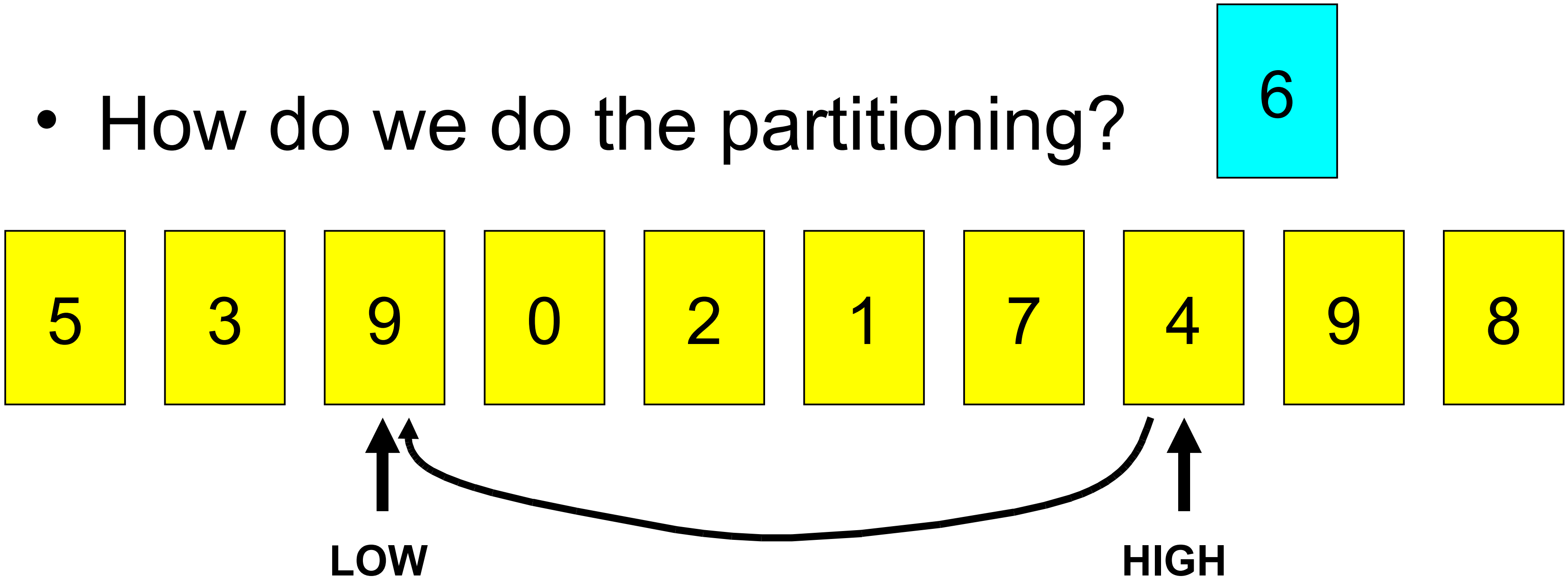
Nitty Gritty Details

- How do we do the partitioning?



- The number at position HIGH is moved to position LOW.

Nitty Gritty Details

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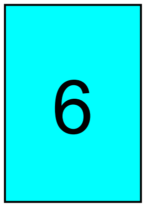

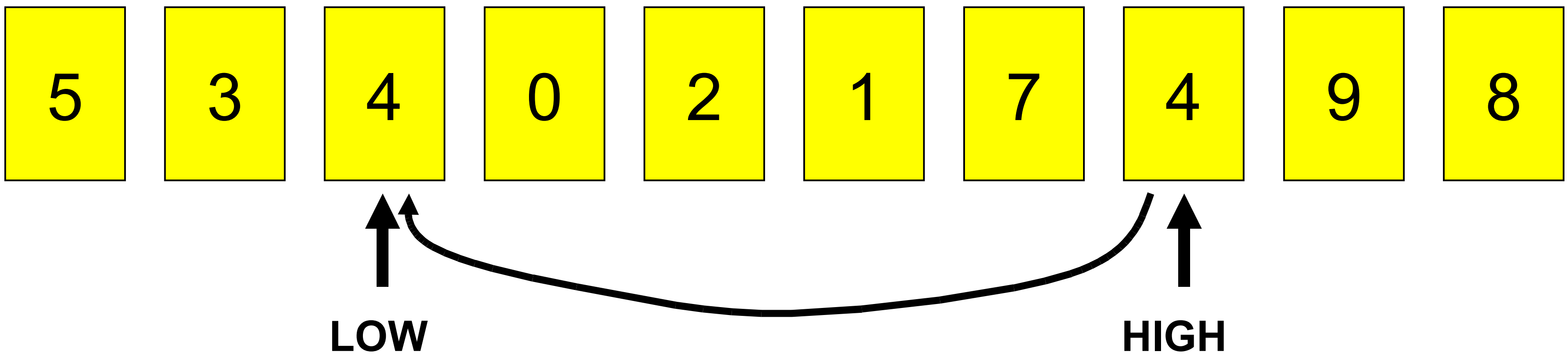

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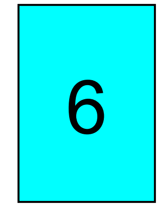

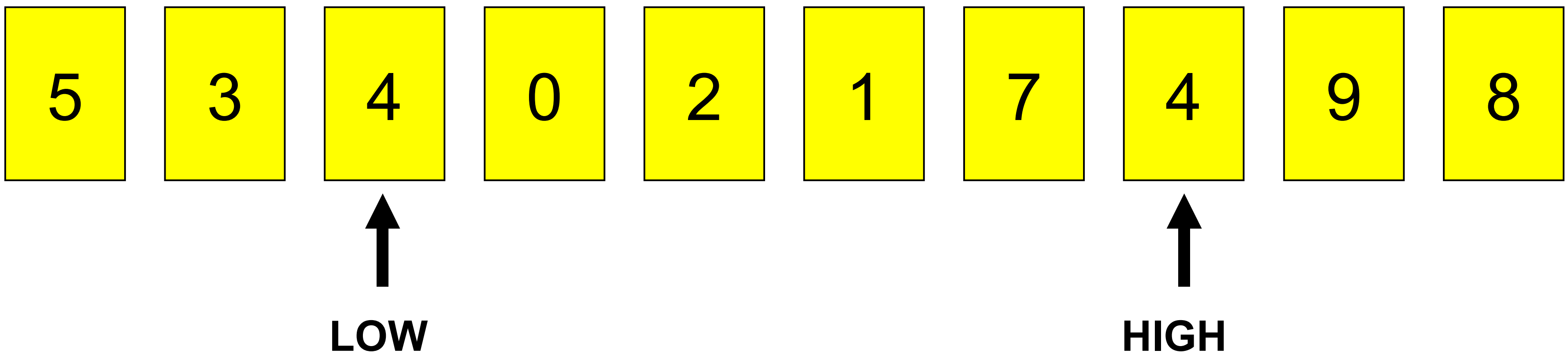

- LOW marches right until it finds a number larger than PIVOT.

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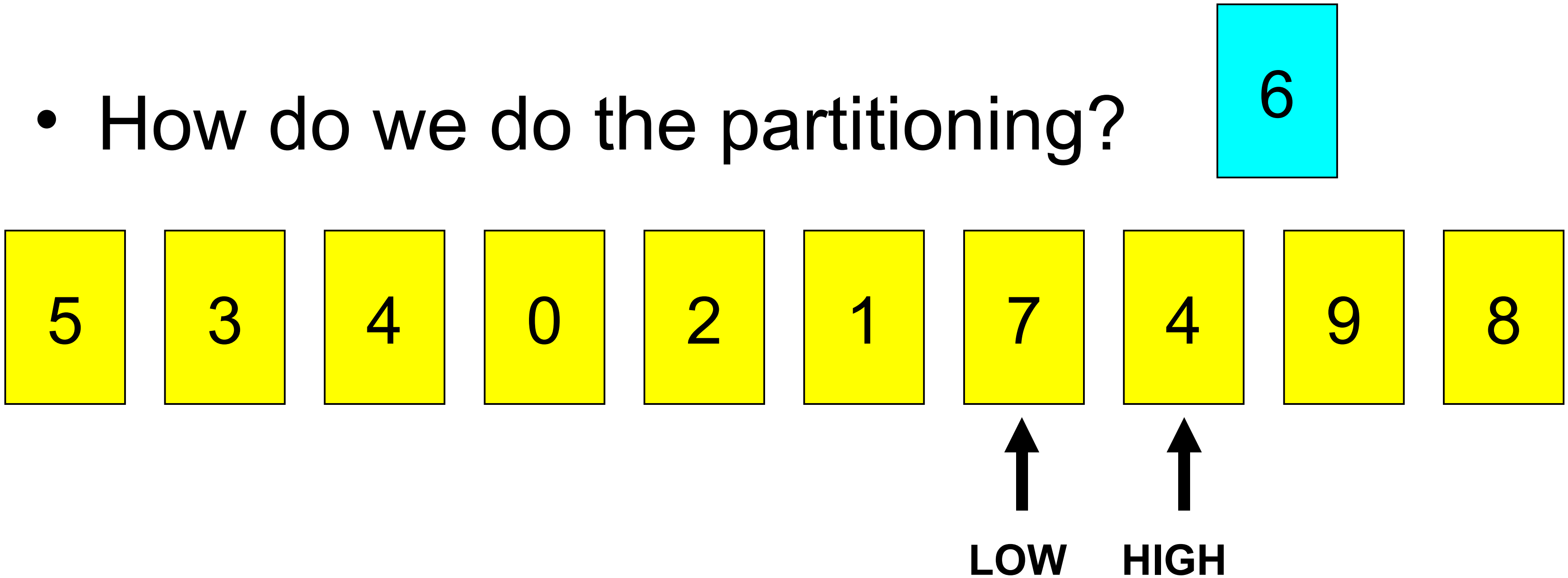

- LOW marches right until it finds a number larger than PIVOT.

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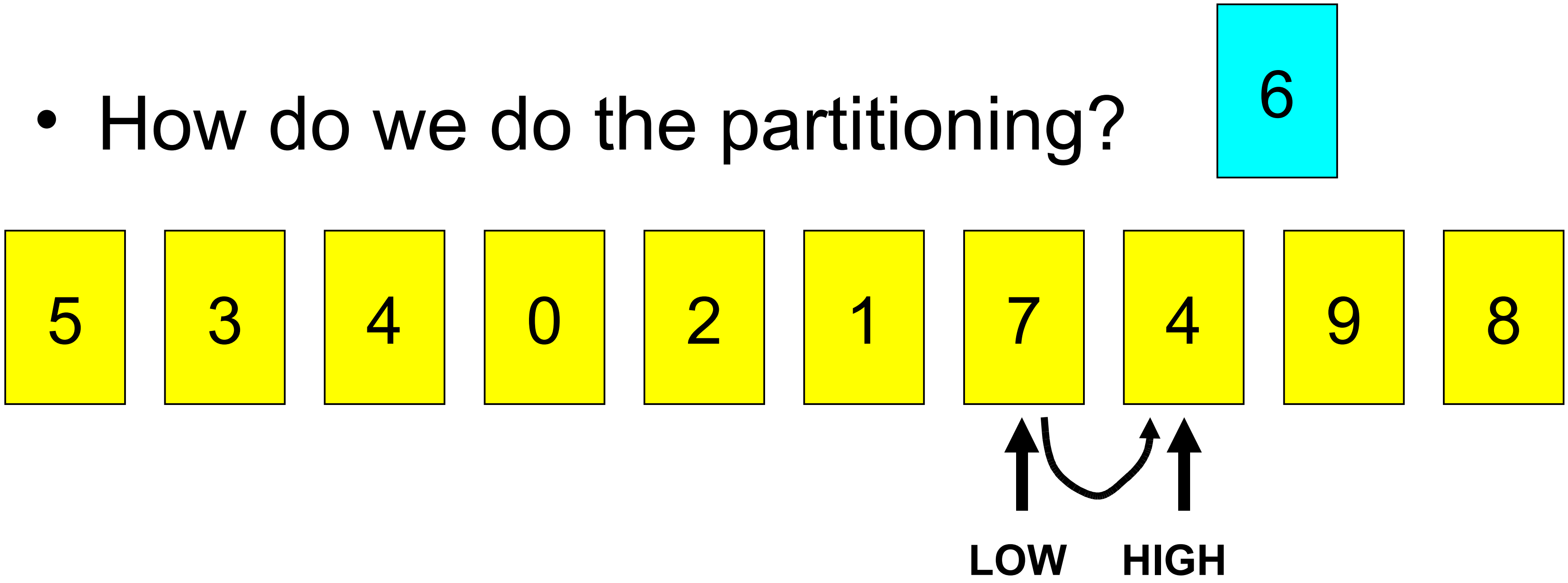

- The number at position LOW is moved to position HIGH.

Nitty Gritty Details

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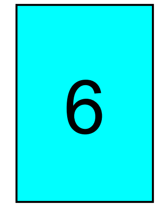

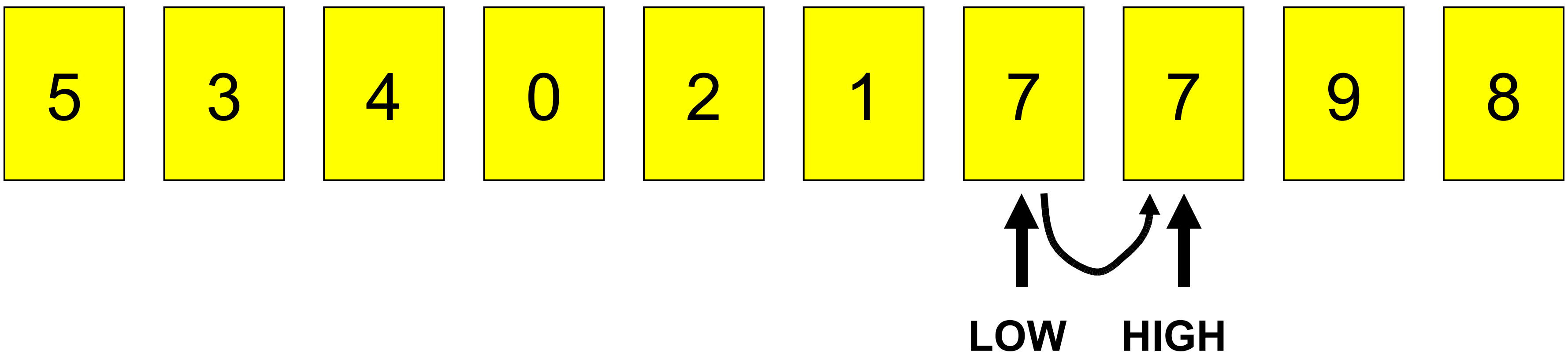

- The number at position LOW is moved to position HIGH.

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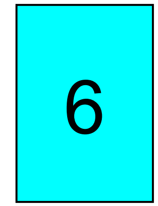
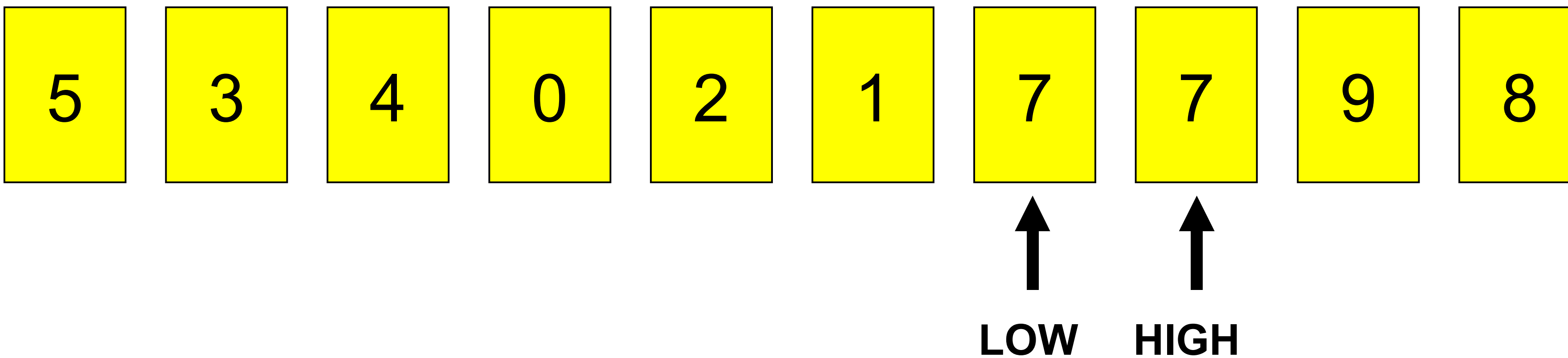
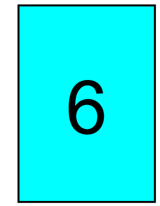


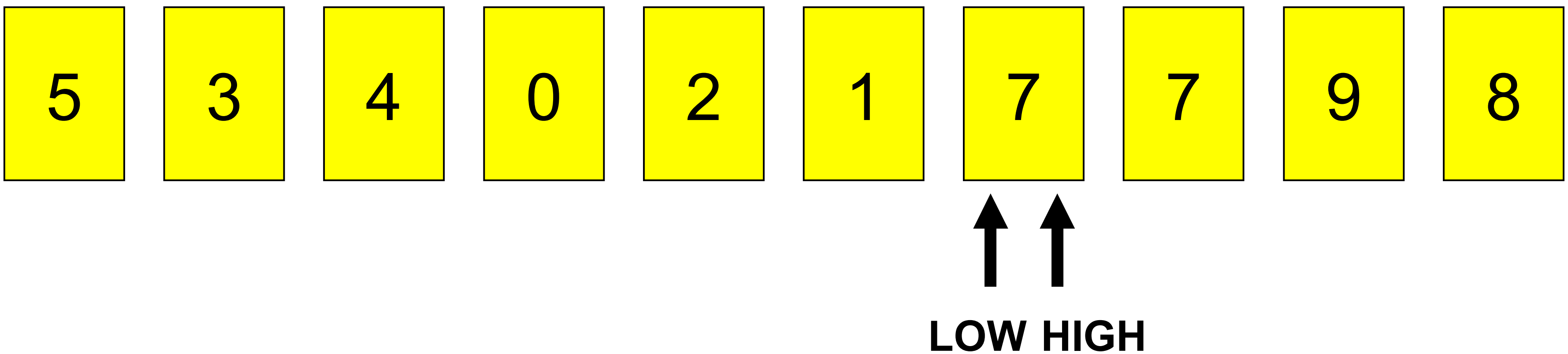


- HIGH marches LEFT until it aligns with LOW.

Nitty Gritty Details

- How do we do the partitioning?





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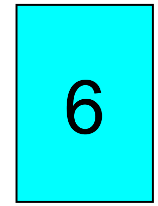
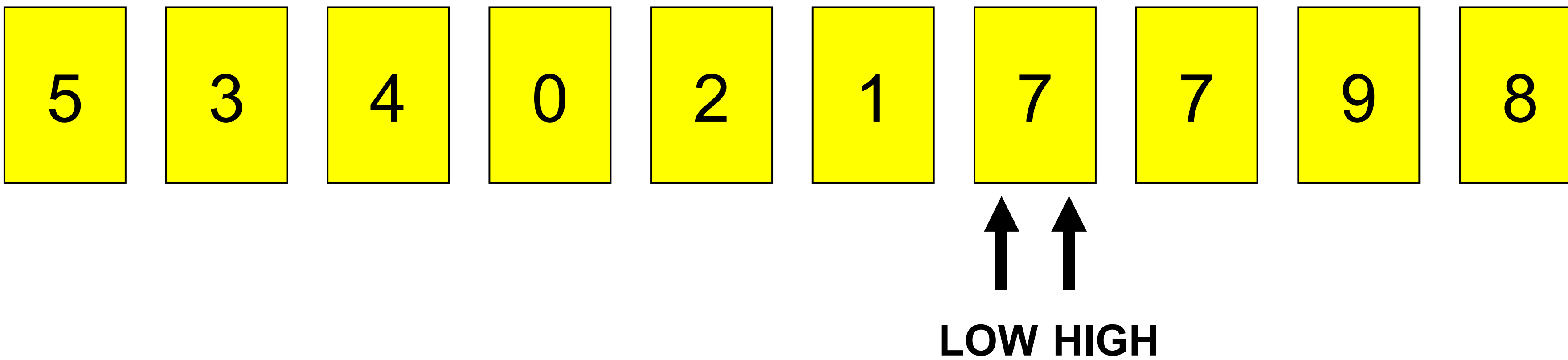
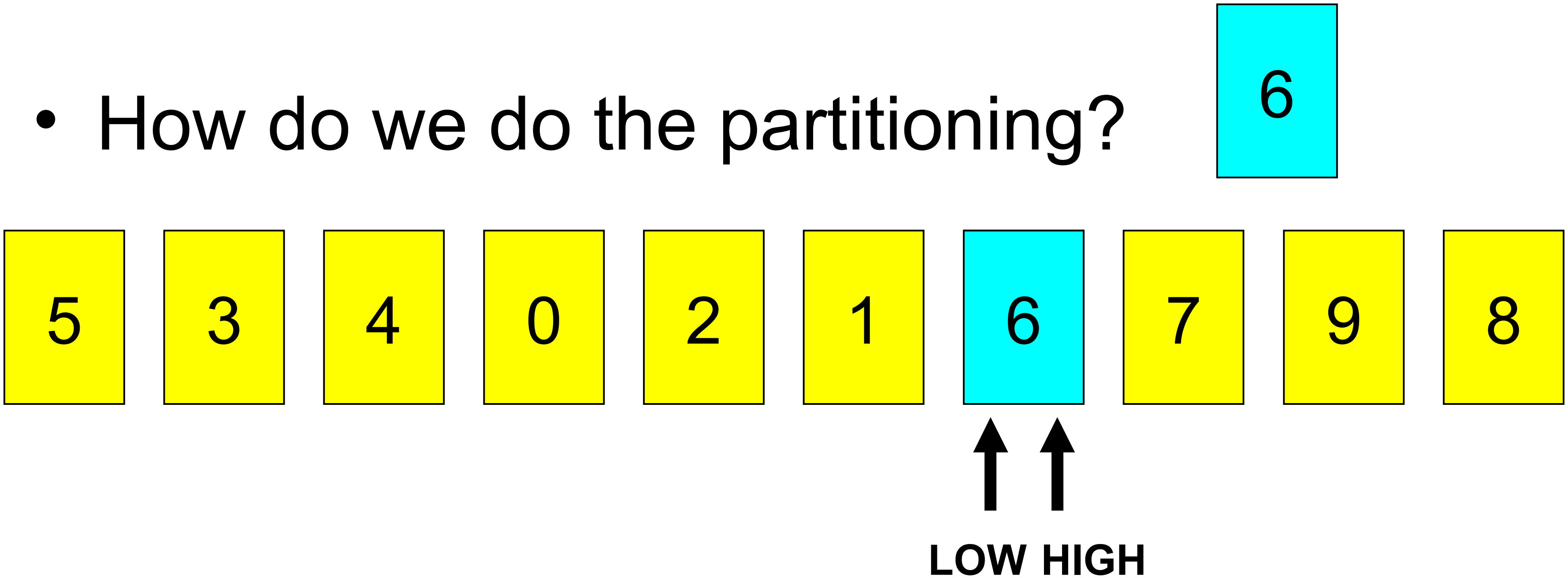




- The PIVOT is inserted at the position LOW/HIGH.

Nitty Gritty Details

- How do we do the partitioning?



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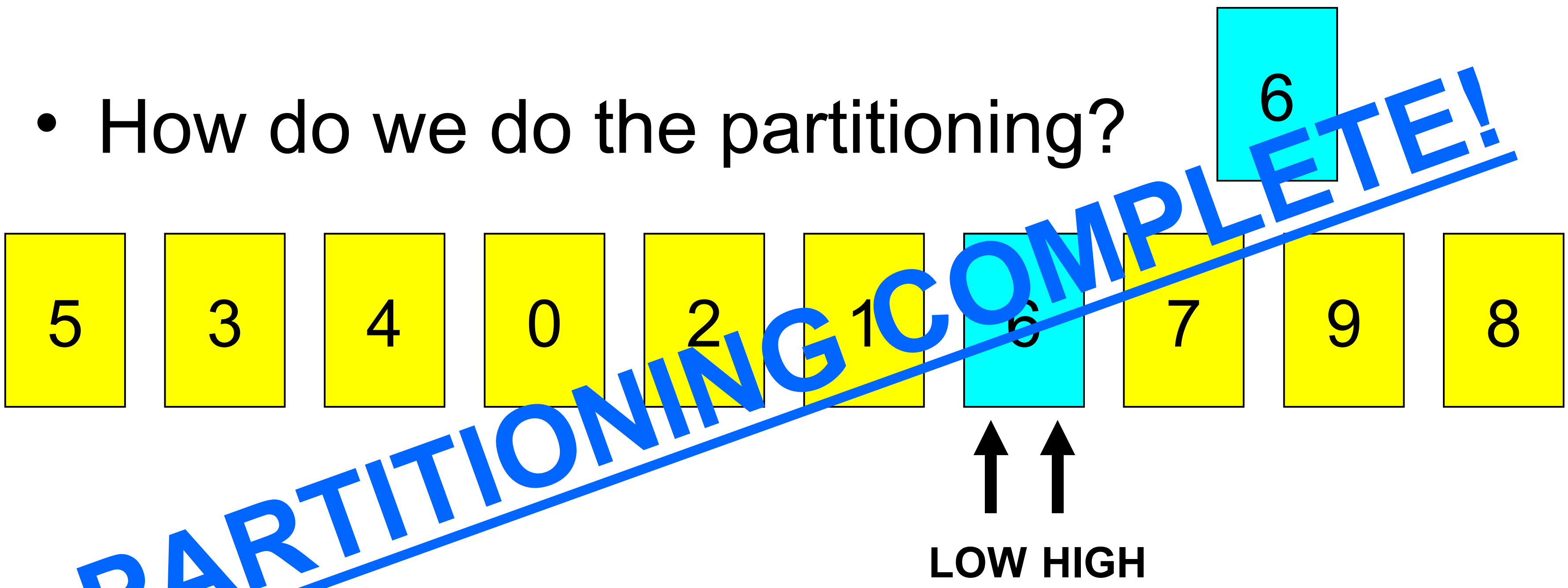

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Quicksort

- How many comparisons/swaps?
  - Sort 1000 items:

1000 items

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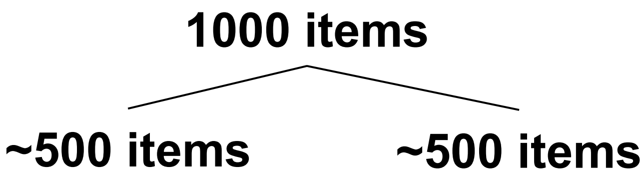

Quicksort

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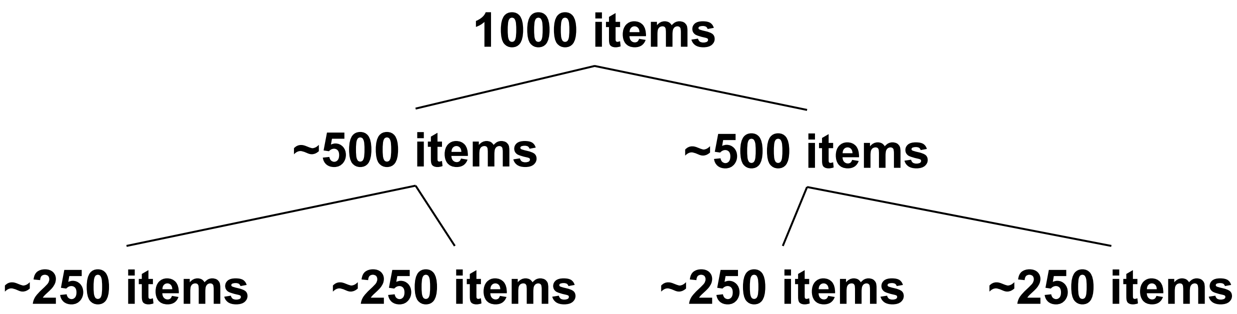

Quicksort

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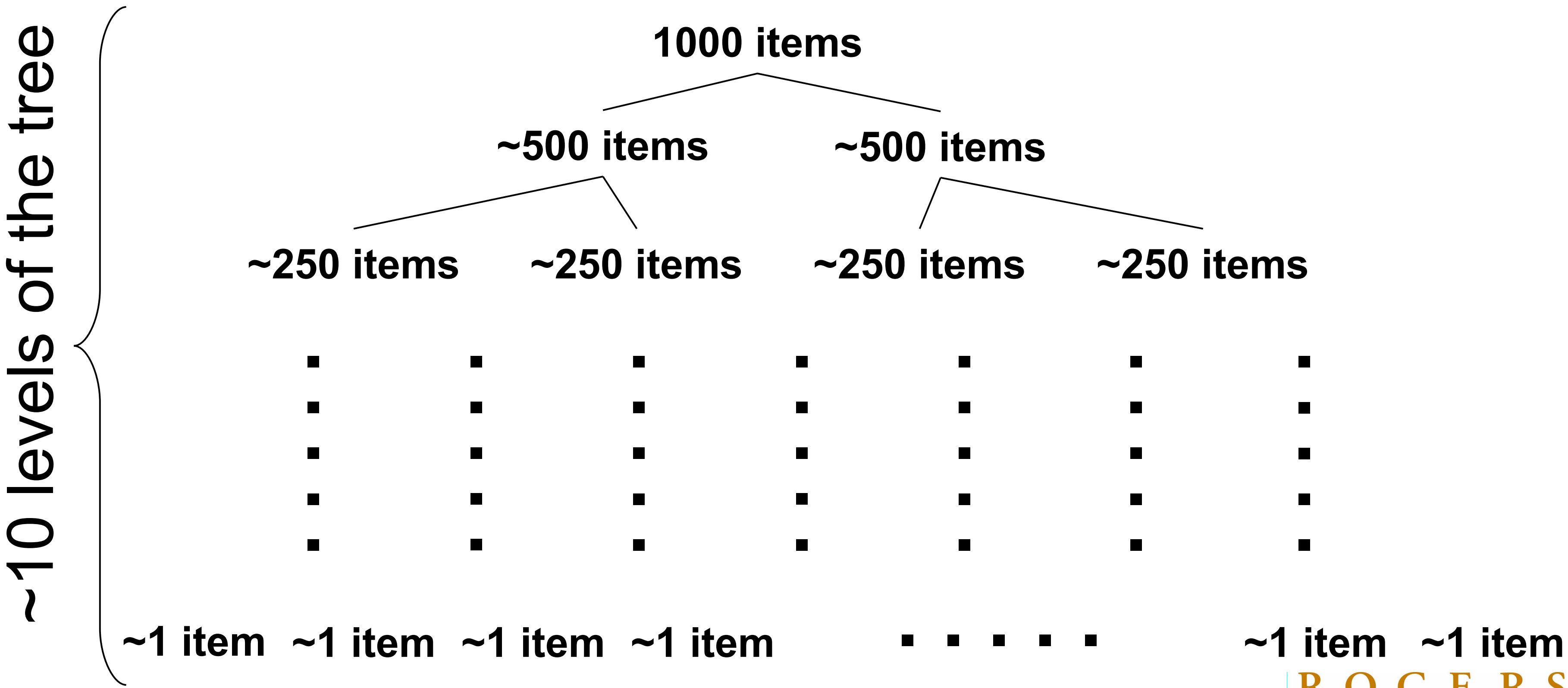

Comparing Algorithms

- To sort 1000 times:
  - Quicksort requires ~10,000 comparisons (on average).
  - Bubble sort may require ~500,000 comparisons.
  - Insertion sort may require ~500,000 comparisons.

Sorting Summary

- **Sorting:** key part of many computer programs!!
- **Algorithm:** set of steps for completing a task.
- Different algorithms for same task may have different efficiencies!
- Talked about three sorting algorithms:
  - **Bubble sort, Insertion Sort** and **Quicksort**
- Bubble sort and insertion sort are simple to build, but need **many steps** to sort the list.
- Quicksort is more complex to build, sorts list **much faster**.

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